

6.003: Signals and Systems—Fall 2003

PROBLEM SET 9

Issued: November 18, 2003

Due: November 26, 2003

Reading Assignments:

Lectures #18-20 & PS#9: Chapters 9 & 11 (through Subsection 11.3.4) of O&W

Lectures #21-22 & PS#10: Chapters 10 & 11 (through Subsection 11.3.4) of O&W

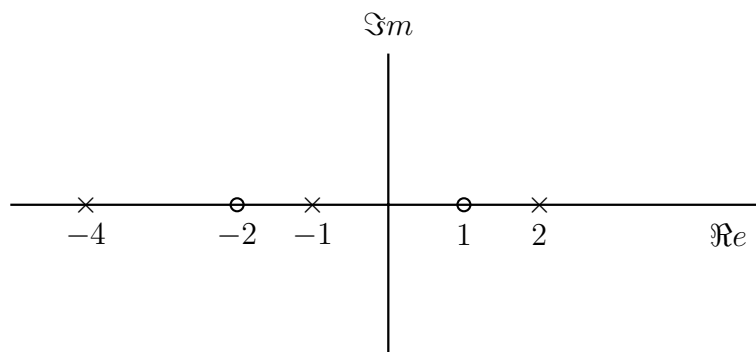
Exercise for home study (not to be turned in, although we will provide solutions):

O&W 9.25 (e)

O&W 9.40

Problems to be turned in:

Problem 1 Consider an LTI system for which the system function $H(s)$ is rational and has the pole-zero pattern shown below:

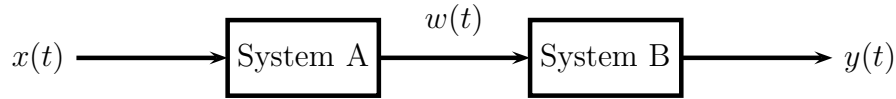


- Indicate all possible ROC's that can be associated with this pole-zero pattern.
- For each ROC identified in Part (a), specify whether the associated system is stable and/or causal.

Problem 2 Draw a direct-form representation for the causal LTI system with system function

$$H(s) = \frac{s(s+1)}{(s+3)(s+4)}.$$

Problem 3 Consider the cascade of two LTI systems as depicted below:



where we have the following:

- System A is causal with impulse response

$$h(t) = e^{-2t}u(t)$$

- System B is causal and is characterized by the following differential equation relating its input, $w(t)$, and output, $y(t)$:

$$\frac{dy(t)}{dt} + y(t) = \frac{dw(t)}{dt} + \alpha w(t)$$

- If the input $x(t) = e^{-3t}$, the output $y(t) = 0$.
- (a) Find the system function $H(s) = Y(s)/X(s)$, determine its ROC and sketch its pole-zero pattern. Note: Your answer should only have numbers in them (i.e., you have enough information to determine the value of α).
- (b) Determine the differential equation relating $y(t)$ and $x(t)$.

Problem 4 Suppose we are given the following information about a causal and stable LTI system with impulse response $h(t)$ and a rational function $H(s)$:

- The steady state response to a unit step, i.e., $s(\infty) = \frac{1}{3}$.
- When the input is $e^t u(t)$, the output is absolutely integrable.
- The signal

$$\frac{d^2h(t)}{dt^2} + 5\frac{dh(t)}{dt} + 6h(t)$$

is of finite duration.

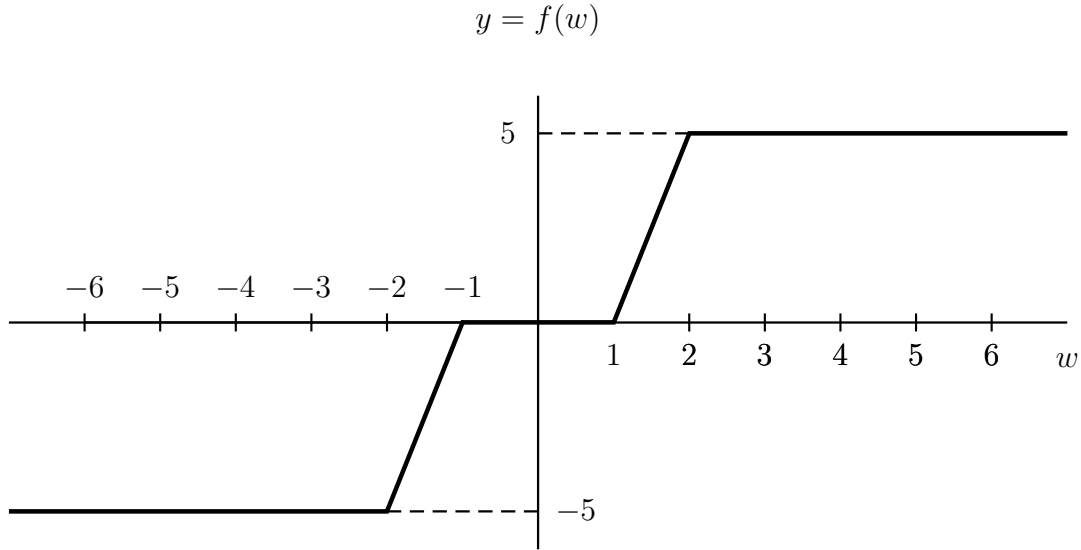
- $h(t)$ has exactly one zero at infinity.

Determine $H(s)$ and its ROC.

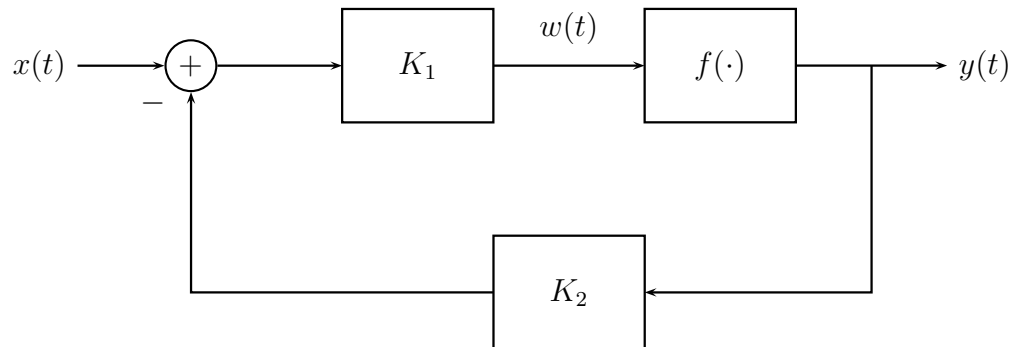
Problem 5 Consider the basic feedback system of Figure 11.3 (a) on p.819 of O&W. Determine the closed-loop system impulse response when

$$H(s) = \frac{1}{s+5}, \quad G(s) = \frac{2}{s+2}.$$

Problem 6 Consider a system whose output, $y(t)$, is characterized by a memoryless non-linear function of its input, $w(t)$ as shown below:

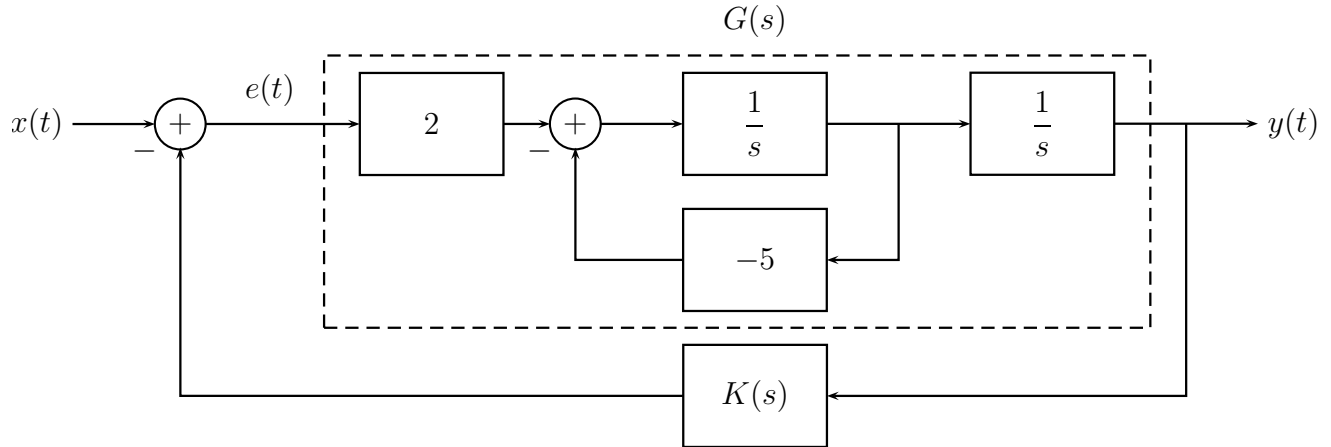


We can see that the function $f(w(t)) = y(t)$ has a deadzone when the magnitude of the input $w(t)$ is less than unity and saturates when the magnitude of the input exceeds 2. In this problem, we would like to see how to reduce this nonlinearity by feedback. Consider the following feedback system:



- (a) Sketch the inverse function $f^{-1}(y)$. Clearly label the axes and indicate the important numbers on the sketch.
- (b) Express $x(t)$ as a linear combination of $y(t)$ and $f^{-1}(y)$.
- (c) Sketch y as a function of x when
- (c.1) $K_1 = 0.5$ and $K_2 = 5$.
- (c.2) $K_1 = 10$ and $K_2 = 0.1$.
- (d) In order to have an approximately linear relation between the input $x(t)$ and the output $y(t)$, are the magnitudes of the gains K_1 and K_2 large? Explain.

Problem 7 Consider the following feedback system.



- (a) Find the system function $G(s) = \frac{Y(s)}{E(s)}$ which is contained in the dashed box in the figure above. Is this system stable?
- (b) Suppose $K(s) = K_p$ where K_p is a real constant. Can you find a range of K_p such that the closed loop system $H(s) = \frac{Y(s)}{X(s)}$ is stable? If so, find the range of K_p .
- (c) Suppose $K(s) = K_d s + K_p$ where both K_d and K_p are real constants. Can you find ranges of K_d and K_p such that the closed loop system $H(s)$ is stable? If so, find the ranges of K_d and K_p .

Reminder: The first 20 problems in each chapter of O&W have answers included at the end of the text. Consider using these for additional practice, either now or as you study for tests.