MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Fall 2003

Problem Set 7 Solution

Exercise for home study

O&W 7.28

(a) Using the Fourier series coefficients of $x(t)$, we can write:

$$x(t) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|k|} e^{jkw_0 t},$$

where

$$w_0 = \frac{2\pi}{0.1 \text{ sec}} = 20\pi \text{ rad/sec}.$$

The lowpass filter $H(jw)$ has a cutoff frequency $w_c = 205\pi$ rad/sec. Thus, $x_c(t)$ is $x(t)$ where all terms with frequency above w_c are removed by the lowpass filter. The terms which are kept have $|kw_0| \leq 205\pi$ rad/sec $\Longrightarrow |k| \leq 10.25$, so the output, $x_c(t)$, is

$$x_c(t) = \sum_{k=-10}^{10} \left(\frac{1}{2}\right)^{|k|} e^{jkw_0 t}$$

To obtain $x[n]$, we sample $x_c(t)$ every $T = 5 \times 10^{-3}$ seconds with an impulse train. The sampling frequency is $\frac{2\pi}{T} = 400\pi = 2 \times$ maximum frequency in $x_c(t)$. Therefore, we can write,

$$\begin{aligned} x[n] &= x_c(nT) \\ &= \sum_{k=-10}^{10} \left(\frac{1}{2}\right)^{|k|} e^{jkw_0 (nT)} \\ &= \sum_{k=-10}^{10} \left(\frac{1}{2}\right)^{|k|} e^{jkw_D n}, \quad \text{where } w_D = w_0 T = 0.1\pi \text{ rad.} \qquad (1) \end{aligned}$$

Note that the complex discrete-time exponentials $e^{jkw_D n}$ are all periodic with period $N = 2\pi/w_D = 20$ (although N is not the fundamental period for all of them). Hence $x[n]$ must also be periodic with period $N = 20$.

(b) To find the Fourier series representation for $x[n]$, we rewrite Equation (1) in the form of a Fourier series synthesis equation:

$$\sum_{k=-10}^{10} \left(\frac{1}{2}\right)^{|k|} e^{jkw_D n} \quad = \sum_{k=<20>} a_k e^{jkw_D n} \tag{2}$$

Note that there are 21 terms ($k = -10...10$) in the left-hand side of Equation (2), but there are only 20 terms in the Fourier series on the right-hand side (as $x[n]$ is periodic with $N = 20$). Looking at the $k = 10$ and $k = -10$ terms:

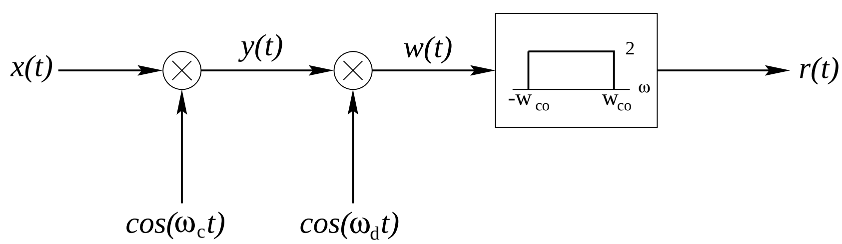
$$\begin{aligned} k = 10: &\quad \left(\tfrac{1}{2}\right)^{10} e^{j10(0.1\pi)n} = \left(\tfrac{1}{2}\right)^{10} (e^{j\pi})^n = \left(\tfrac{1}{2}\right)^{10} (-1)^n \\ k = -10: &\quad \left(\tfrac{1}{2}\right)^{10} e^{j(-10)(0.1\pi)n} = \left(\tfrac{1}{2}\right)^{10} (e^{-j\pi})^n = \left(\tfrac{1}{2}\right)^{10} (-1)^n \end{aligned}$$

We can add these two terms since they involve the same complex exponential, $(-1)^n$. Now we have the full set of Fourier series coefficients for $x[n]$. The coefficients are periodic with period $N = 20$, and over the period $-9 \leq k \leq 10$ have values

$$a_k \quad = \quad \begin{cases} \left(\frac{1}{2}\right)^{|k|}, & |k| \leq 9 \\ 2\left(\frac{1}{2}\right)^{10}, & k = 10. \end{cases}$$

O&W 8.23

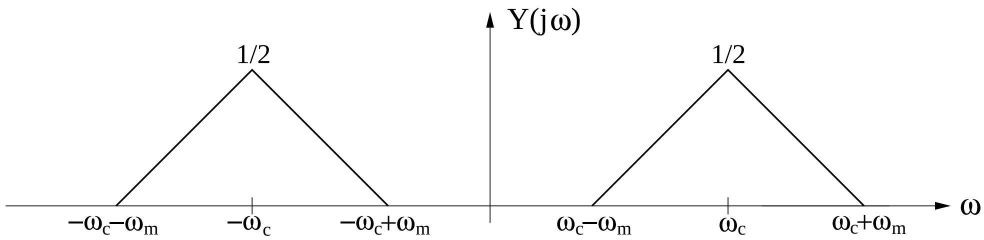
(a) The block diagram of modulation and demodulation for this problem is shown below:



We want to show that the output of the lowpass filter in the demodulator is proportional to $x(t)\cos(\Delta\omega t)$, where $\Delta\omega = \omega_d - \omega_c$. We show this algebraically as well as graphically for the example in Figure P8.23. $y(t)$ is just $x(t)$ multiplied by $\cos(\omega_c t)$. Using the multiplication property, $Y(j\omega)$ consists of two shifted copies of $X(j\omega)$, centered at $\omega = \omega_c$ and $\omega = -\omega_c$, with amplitude scaled by 1/2:

$$Y(j\omega) \quad = \quad \frac{1}{2}\left[X(j(\omega - \omega_c)) + X(j(\omega + \omega_c))\right]$$

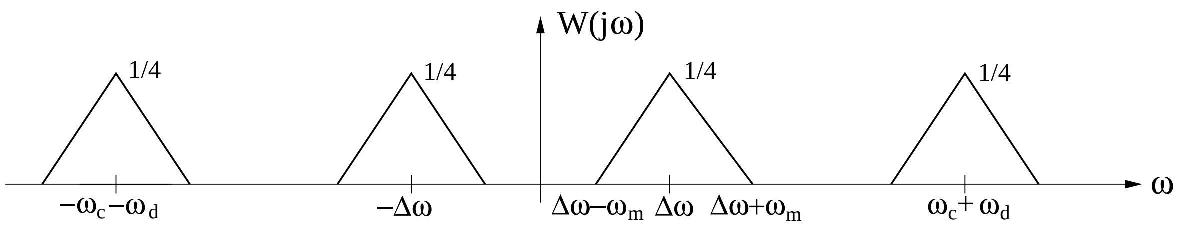
$Y(j\omega)$ is sketched below:



Similarly, $W(j\omega)$ will consist of two shifted and scaled copies of $Y(j\omega)$:

$$\begin{aligned} W(j\omega) &= \frac{1}{2}\left[Y(j(\omega-\omega_d)) + Y(j(\omega+\omega_d))\right] \\ &= \frac{1}{4}\left[X(j(\omega-\omega_d-\omega_c)) + X(j(\omega-\omega_d+\omega_c)) + X(j(\omega+\omega_d-\omega_c))\right. \\ &\quad \left. + X(j(\omega+\omega_d+\omega_c))\right] \\ &= \frac{1}{4}\left[X(j(\omega-\omega_c-\omega_d)) + X(j(\omega-\Delta\omega)) + X(j(\omega+\Delta\omega)) + X(j(\omega+\omega_c+\omega_d))\right] \end{aligned}$$

If we assume that the two copies centered at $\Delta\omega$ and $-\Delta\omega$ do not overlap, then $W(j\omega)$ will look like the following:



Finally, $R(j\omega)$ is $W(j\omega)$ passed through a lowpass filter with cutoff frequency between $\omega_m + \Delta\omega$ and $2w_c + \Delta w - w_m = \omega_c + \omega_d - \omega_m$. This filtering process removes the high frequency content of $W(j\omega)$ so that only the two triangles at low frequencies remain. Their amplitudes are scaled by 2, since the gain of the lowpass filter is 2. Therefore, $R(j\omega)$, the output of the demodulator, will be

$$R(j\omega) = \frac{1}{2}\left[X(j(\omega-\Delta\omega)) + X(j(\omega+\Delta\omega))\right]$$

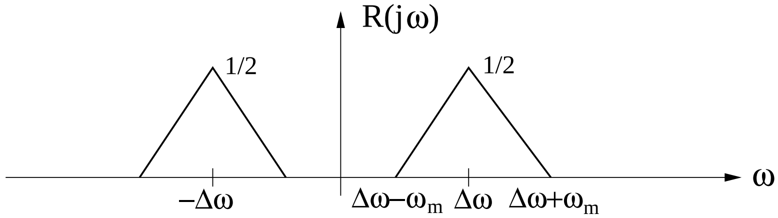
which in the time domain gives us

$$r(t) = x(t)\cos(\Delta\omega t)$$

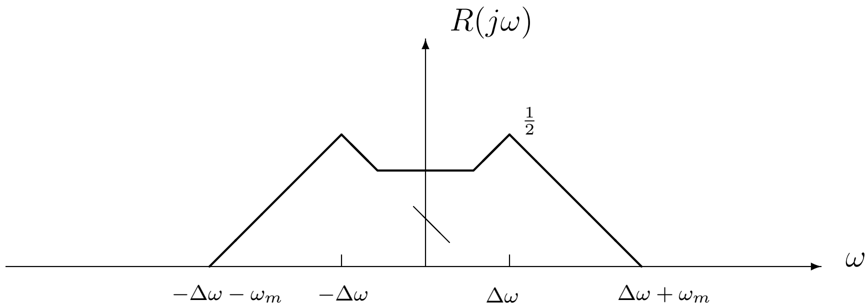
(b) From part (a), the spectrum of the output of the demodulator will be

$$R(j\omega) = \frac{1}{2}\left[X(j(\omega-\Delta\omega)) + X(j(\omega+\Delta\omega))\right]$$

If the two copies of $X(j\omega)$ do not overlap, as shown in the figures above, this will look like:

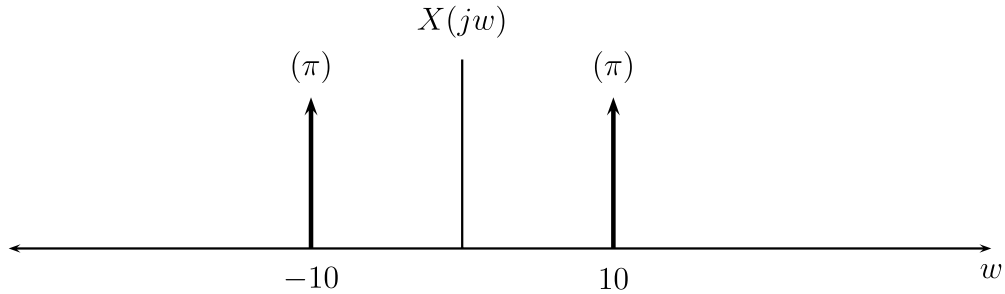


This will happen if $\Delta\omega \geq \omega_M$. If this is not true, then the two copies will overlap, and the output will look like:

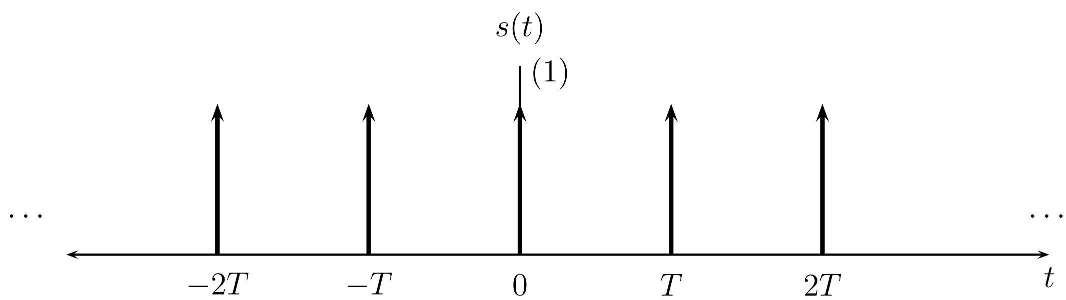


Problem 1

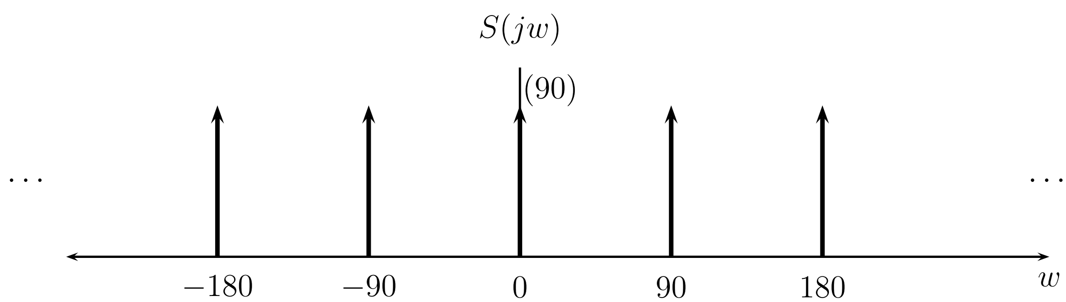
(a) We are given $x(t) = \cos(10t)$. Here, $\omega_o = 10$ rad/sec. Taking the Fourier transform of $x(t)$,



The sampling function, $s(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT)$, with $T = \frac{2\pi}{90}$.



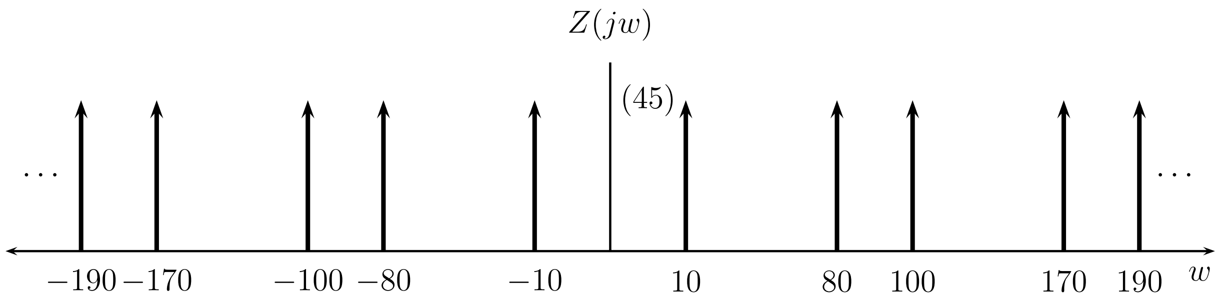
Taking the Fourier transform of $s(t)$ (note that $\omega_s = \frac{2\pi}{T} = 90$),



Using the multiplication property, $z(t) = x(t)s(t)$ in frequency domain is $Z(jw) = \frac{1}{2\pi}(X(jw) * S(jw))$, i.e. we need to convolve $X(jw)$ with the periodic impulse train in $S(jw)$ and scale the amplitude by $\frac{1}{2\pi}$ (see section 7.1.1 in O&W).

$$\begin{aligned} z(t) &= \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT) \\ Z(jw) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\theta) S(j(w - \theta)) d\theta \end{aligned}$$

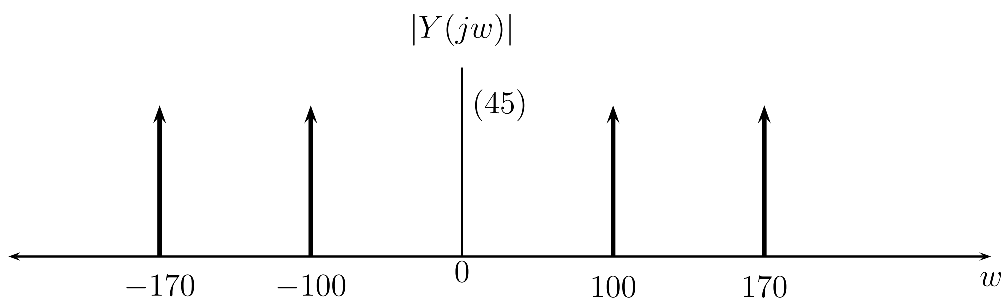
Therefore, $Z(jw)$ is as follows:



(b) $y(t)$ is the output from the band-pass filter, $H(jw)$, with input $z(t)$ as derived in part (a). We know,

$$Y(jw) = H(jw)Z(jw)$$

Let us consider $|Y(jw)|$ and $\angle Y(jw)$ separately. $|Y(jw)|$ is the band-pass filtered version of $|Z(jw)|$ with frequency components between 90 to 180 and -180 to -90 rad/sec.

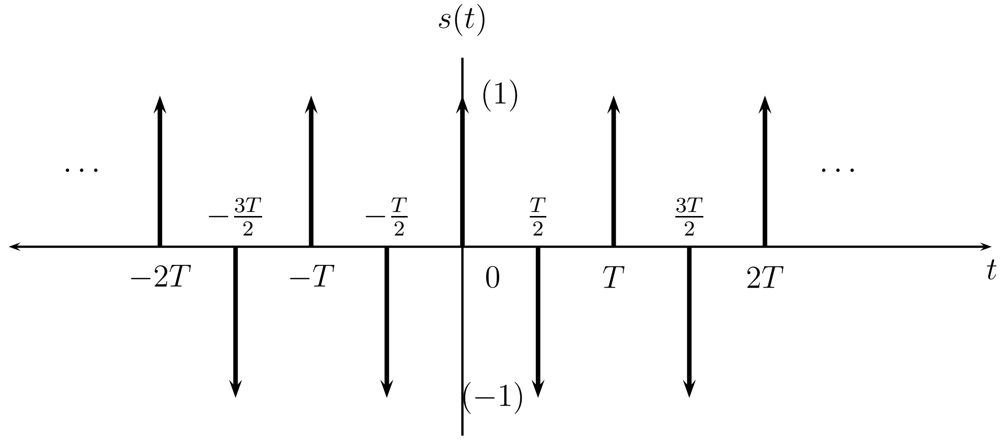


$$\begin{aligned}\angle Y(jw) &= \angle H(jw) + \angle Z(jw) \\ &= -\frac{\pi w}{200} + 0 = -\frac{\pi w}{200}\end{aligned}$$

Combining the magnitude and angle, $Y(jw) = |Y(jw)|e^{j\angle Y(jw)}$.
Consider $Y(jw)$ as the Fourier transform of the sum of two sinusoidal signals; one with $w_o = 100$ and another with $w_o = 170$. Using the time-shifting property of Fourier transform, $x(t - t_o) \overset{\mathcal{FT}}{\longleftrightarrow} e^{-jwt_o}X(jw)$,

$$\begin{aligned}y(t) &= \frac{45}{\pi}\cos(100(t - \frac{\pi}{200})) + \frac{45}{\pi}\cos(170(t - \frac{\pi}{200})) \\ &= \frac{45}{\pi}\cos(100t - \frac{\pi}{2}) + \frac{45}{\pi}\cos(170t - \frac{17\pi}{20})\end{aligned}$$

(c) Now the sampling function $s(t)$ is changed with $T = \frac{2\pi}{90}$,



$$s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT) - \sum_{k=-\infty}^{\infty} \delta(t - kT - \frac{T}{2})$$

Taking the Fourier transform,

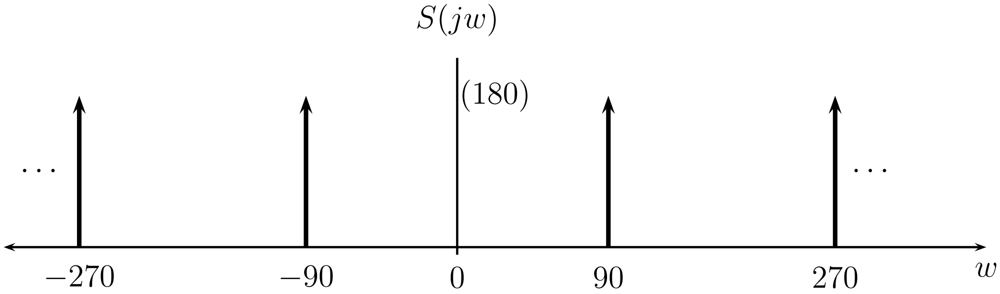
$$\begin{aligned} S(jw) &= \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(w - k\frac{2\pi}{T}) - \frac{2\pi}{T} e^{-jw\frac{T}{2}} \sum_{k=-\infty}^{\infty} \delta(w - k\frac{2\pi}{T}) \\ &= \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(w - k\frac{2\pi}{T}) - \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} e^{-jk\frac{2\pi}{T}\frac{T}{2}} \delta(w - k\frac{2\pi}{T}) \\ &= \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(w - k\frac{2\pi}{T}) - \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} (e^{-j\pi})^k \delta(w - k\frac{2\pi}{T}) \\ &= \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(w - k\frac{2\pi}{T}) - \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} (-1)^k \delta(w - k\frac{2\pi}{T}) \end{aligned}$$

Seperating the odd and even terms of k,

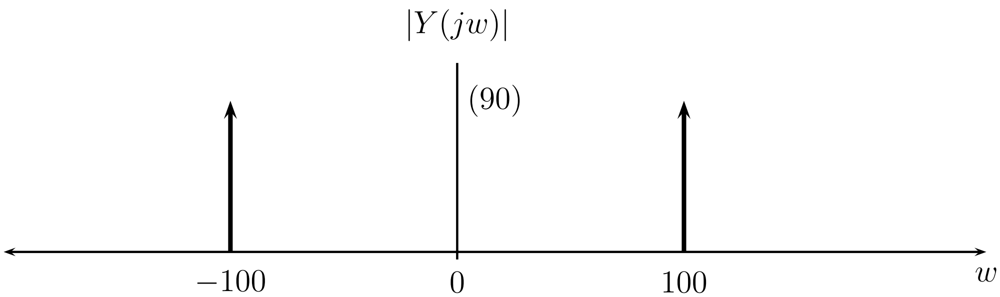
$$\begin{aligned} S(jw) &= \frac{2\pi}{T} \sum_{k=even} \delta(w - k\frac{2\pi}{T}) - \frac{2\pi}{T} \sum_{k=even} \delta(w - k\frac{2\pi}{T}) \\ &\quad + \frac{2\pi}{T} \sum_{k=odd} \delta(w - k\frac{2\pi}{T}) + \frac{2\pi}{T} \sum_{k=odd} \delta(w - k\frac{2\pi}{T}) \\ &= \frac{4\pi}{T} \sum_{k=odd} \delta(w - k\frac{2\pi}{T}) \end{aligned}$$

$x(t) = \cos(10t)$ as before. To find $Z(jw)$, we need to convolve $X(jw)$ with the impulse train in $S(jw)$ and scale the result by $\frac{1}{2\pi}$.

$S(jw)$ is as sketched below,



The convolution will place two scaled impulses (from $X(jw)$) centered at each impulse in the impulse train of $S(jw)$. Finally, $H(jw)$ will only pass impulses that exist between 90 to 180 and -180 to -90 radians. We plot $|Y(jw)|$ (output from $H(jw)$) as follows:



As derived in part (b), $\angle Y(jw) = \angle H(jw) = -\frac{\pi w}{200}$.
From the plot of $|Y(jw)|$ and the $\angle Y(jw)$, we can view $y(t)$ as a time-shifted cos functions. Therefore,

$$
\begin{aligned}
y(t) &= \frac{90}{\pi}\cos(100(t - \frac{\pi}{200})) \\
&= \frac{90}{\pi}\cos(100t - \frac{\pi}{2})
\end{aligned}
$$

Problem 2 (O&W 7.30 except let $x_c(t) = \delta(t - \frac{T}{2})$)

(a) We are given $x_c(t)$

$$\begin{aligned} x_c(t) &= \delta(t - \frac{T}{2}) \\ X_c(jw) &= \mathrm{e}^{-jw\frac{T}{2}} \end{aligned}$$

We take the Fourier transform of the system's differential equation and find the frequency response, $H(jw)$, of the system.

$$\begin{aligned} \frac{dy_c(t)}{dt} + y_c(t) &= x_c(t) \\ jwY_c(jw) + Y_c(jw) &= X_c(jw) \\ H(jw) = \frac{Y_c(jw)}{X_c(jw)} &= \frac{1}{1+jw} \end{aligned}$$

Now, we can write,

$$\begin{aligned} Y_c(jw) &= X_c(jw)H(jw) = \mathrm{e}^{-jw\frac{T}{2}}\frac{1}{1+jw} \\ y_c(t) &= e^{-(t-\frac{T}{2})}u(t - \frac{T}{2}) \end{aligned}$$

(b) $y[n] = y_c(nT)$ where $y_c(t)$ is as defined in part (a). Therefore, $y_c(nT)$ will pick-up values from $y_c(t)$ at nT time values with $n = 0, 1, 2, ...$

$$\begin{aligned} y[n] = y_c(nT) &= e^{-nT+\frac{T}{2}}u[n-1] \\ &= (e^{\frac{T}{2}})(e^{-T})(e^{-T})^{n-1}u[n-1] \end{aligned}$$

Using the time-shifting property of DTFT and basic DTFT table,

$$Y(e^{jw}) = e^{-\frac{T}{2}}e^{-jw}\frac{1}{1-e^{-T}e^{-jw}}$$

Now we choose $H(e^{jw})$ such that:

$$\begin{aligned} y[n] * h[n] &= w[n] = \delta[n] \\ Y(e^{jw})H(e^{jw}) &= 1 \\ H(e^{jw}) &= \frac{1}{e^{-\frac{T}{2}}e^{-jw}}(1 - e^{-T}e^{-jw}) \\ H(e^{jw}) &= e^{\frac{T}{2}}e^{jw} - e^{-\frac{T}{2}} \end{aligned}$$

Taking the inverse FT,

$$h[n] = e^{\frac{T}{2}}\delta[n+1] - e^{-\frac{T}{2}}\delta[n]$$

Problem 3

First, we need to find frequency response of the DT filter, $y[n] = \frac{3}{4}y[n-2]+x[n]+\frac{1}{4}x[n-1]$ When $x[n] = \delta[n]$, $y[n] = h[n]$. Therefore,

$$\begin{aligned} h[n] &= \frac{3}{4}h[n-2] + \delta[n] + \frac{1}{4}\delta[n-1] \\ H(e^{j\Omega}) &= \frac{3}{4}e^{-j2\Omega}H(e^{j\Omega}) + 1 + \frac{1}{4}e^{-j\Omega} \\ H(e^{j\Omega}) &= \frac{1+\frac{1}{4}e^{-j\Omega}}{1-\frac{3}{4}e^{-j2\Omega}}, \quad |\Omega| < \pi \end{aligned}$$

It is given that $X(j\omega) = 0$ for $|\omega| \geq \frac{\pi}{T}$ and we have a sampling frequency, $\omega_s = \frac{2\pi}{T}$. So there will be no aliasing.

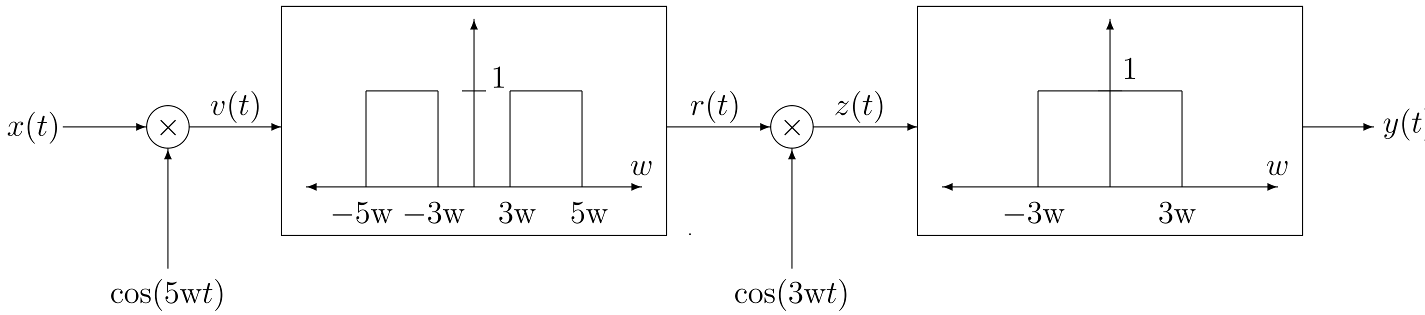
Therefore, the effective frequency response of the entire CT system, $H_c(jw)$, is related to the frequency response of the DT system, $H(e^{j\Omega})$, by (assume $\Omega = wT$ and find appropriate range of w):

$$H_c(j\omega) = \begin{cases} H(e^{j\omega T}), & |w| \leq \frac{\pi}{T} = \frac{\omega_s}{2} \\ 0, & |w| > \frac{\pi}{T} \end{cases}$$

$$H_c(j\omega) = \begin{cases} \frac{1+\frac{1}{4}e^{-j\omega T}}{1-\frac{3}{4}e^{-j2\omega T}}, & |w| \leq \frac{\pi}{T} = \frac{\omega_s}{2} \\ 0, & |w| > \frac{\pi}{T} \end{cases}$$

Problem 4, O&W 8.22

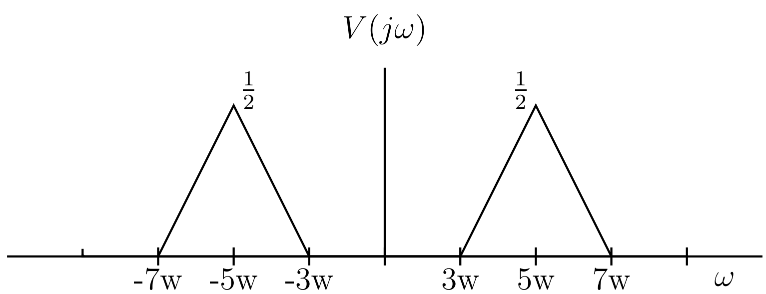
Let us define $v(t), r(t)$, and $z(t)$ as shown below in the system diagram.



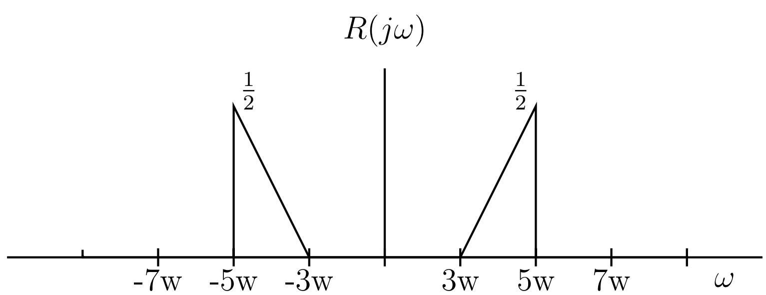
$v(t)$ is the output of sinusoidal amplitude modulation. To find $V(j\omega)$:

$$\begin{aligned} v(t) &= x(t)\cos(5\text{w}t) \\ V(j\omega) &= \frac{1}{2\pi}\left(X(j\omega) * \pi\left[\delta(\omega - 5\text{w}) + \delta(\omega + 5\text{w})\right]\right) \\ &= \frac{1}{2}\left[X(j(\omega - 5\text{w})) + X(j(\omega + 5\text{w}))\right] \end{aligned}$$

Graphically,

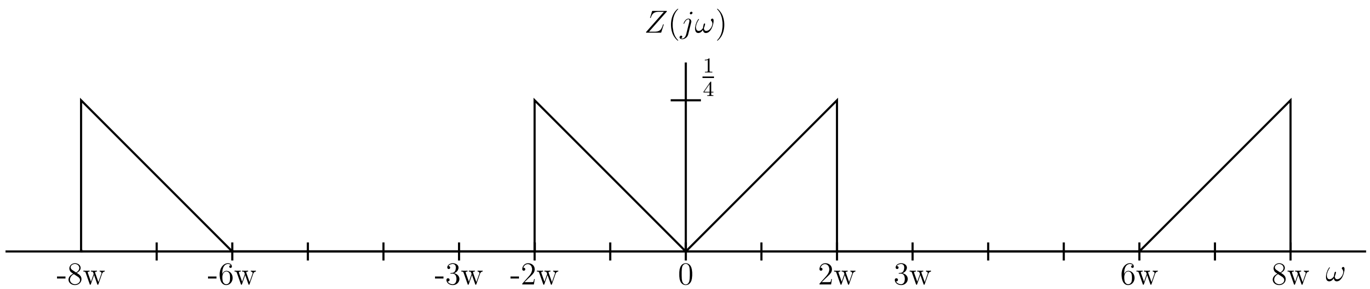


To find $R(j\omega)$, we see that the bandpass filter will only passes frequency components that are between of magnitude 3w to 5w,

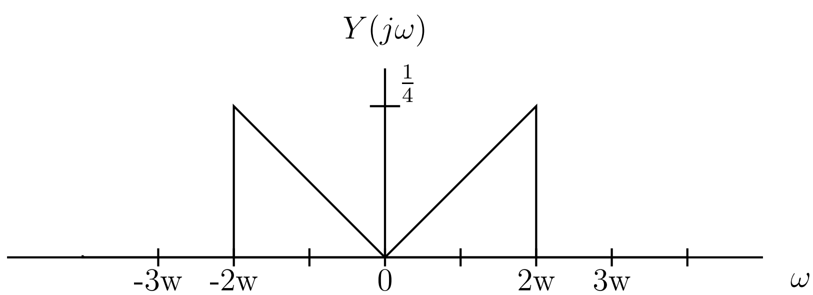


Now $z(t)$ is a sinusoidal amplitude demodulation with a different frequency carrier signal, $\cos(3\text{w}t)$. We can derive $Z(j\omega)$,

$$
\begin{aligned}
z(t) &= r(t)\cos(3\text{w}t) \\
Z(j\omega) &= \frac{1}{2\pi}\left(R(j\omega) * \pi\left[\delta(\omega-3\text{w})+\delta(\omega+3\text{w})\right]\right) \\
&= \frac{1}{2}\left[R(j(\omega-3\text{w}))+R(j(\omega+3\text{w}))\right]
\end{aligned}
$$



Finally, $y(t)$ is $z(t)$ lowpass-filtered, only allowing frequency content from -3w to 3w to go through. Therefore, $Y(j\omega)$ is as follows:

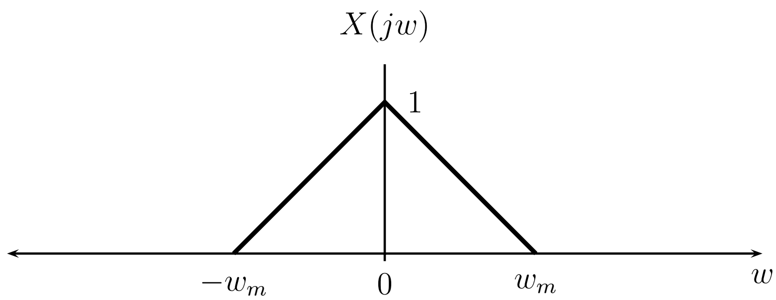


Problem 5 (O&W 8.49)

(a) Let us assume the frequency of the sampling function $s(t)$ is w_c. $s(t)$ is a periodic pulse-train with period T and pulse width, $\Delta = \frac{T}{2}$. Therefore, $w_c = \frac{2\pi}{T}$ and we can write the Fourier series coefficients, a_k, as:

$$\begin{aligned} a_k &= \frac{\sin(kw_c\Delta/2)}{\pi k} \\ &= \frac{\sin(k(2\pi/T)(T/4))}{\pi k} \\ &= \frac{\sin(k\frac{\pi}{2})}{\pi k} \end{aligned}$$

a_k is a periodic sample train with frequency w_c.
Let us assume the input signal, $x(t)$, with Fourier transform, $X(jw)$, as shown below.



Let us define the input of the filter, $H_1(jw)$, to be $z_1(t)$. Therefore, $z_1(t) = x(t)s(t)$ and as discussed in section 8.5.1, $Z_1(jw)$ is the convolution of $X(jw)$ with impulse train with area a_k.

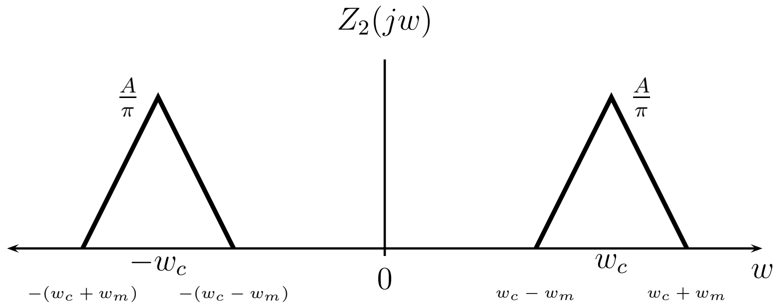
$$Z_1(jw) = \sum_{k=-\infty}^{+\infty} a_k X(j(w - kw_c))$$

In order to prevent aliasing, we need to make sure $X(jw)$ centered at kw_c's in $Z_1(jw)$ does not overlap with another one. $w_c > 2w_m$ will ensure that there is no aliasing or overlap. i.e.

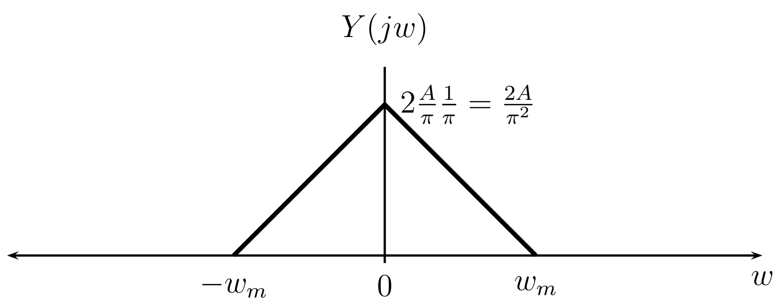
$$\begin{aligned} w_m &< \frac{w_c}{2} \\ \text{highest allowable } w_m &= \frac{2\pi}{T}\frac{1}{2} = \frac{\pi}{T} \end{aligned}$$

If $x(t)$ has frequency greater than $\frac{\pi}{T}$, there will be aliasing and $y(t)$ will not be proportional to $x(t)$.

(b) Let us define the output of the filter, $H_1(jw)$, to be $z_2(t)$. $z_2(t)$ is the bandpass-filtered version of $z_1(t)$. From the pass-band of $H_1(jw)$, we see that $z_2(t)$ will only contain frequency components centered at $\pm(w_c = \frac{2\pi}{T})$ which corresponds to $k = \pm 1$ in $Z_1(jw)$. The magnitude of $Z_1(jw)$ at $k = \pm 1$ is $a_{k=1} = \frac{sin(\pi/2)}{\pi} = \frac{1}{\pi}$. Therefore, $Z_2(jw)$ is as follows :



The second modulation with $s(t)$ will result in convolution of $Z_2(jw)$ with the impulse train with area a_k again. We are only interested in low frequency components as the result is low-pass filtered with $H_2(jw)$. The convolution will cause two images of $\frac{A}{\pi}X(jw)$ from $Z_2(jw)$ to be scaled by $\frac{1}{\pi}$ and superimposed centered at $w = 0$. The low-pass filter, $H_2(jw)$, will only pass frequency components between $-\frac{\pi}{T}$ to $-\frac{\pi}{T}$. Therefore, $Y(jw)$ is as as follows,



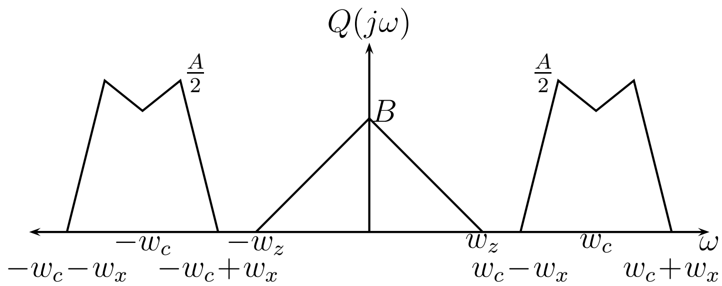
From the plot, we can see $Y(jw)$ has the same shape and bandwidth as $X(jw)$, except that the amplitude is multiplied by $\frac{2A}{\pi^2}$. Therefore, gain of the overall system $= \frac{2A}{\pi^2}$.

Problem 6

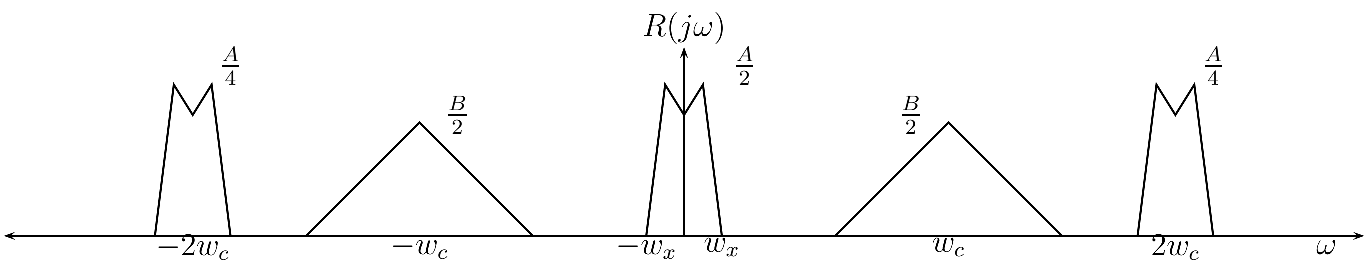
(a) Let's analyze the system graphically in the frequency domain. Consider one step at a time:

(i) $p(t) = x(t)\cos(\omega_c t)$
Multiplying by $\cos(\omega_c t)$ in the time domain means that the Fourier transform of $p(t)$ will be the sum of two $X(jw)$ shifted by ω_c and $-\omega_c$, and magnitude scaled by $\frac{1}{2}$.

(ii) $q(t) = p(t) + z(t)$
The Fourier transform of q(t) is just the sum of the Fourier transforms of p(t) and z(t).



(iii) $r(t) = q(t)\cos(\omega_c t)$
i.e. q(t) is modulated by $\cos(\omega_c t)$. Therefore, repeating the procedure in (i):



(iv) Filter $r(t)$ with $H_{LP}(jw)$.
The baseband copy of $X(j\omega)$ is nonzero only between $-\omega_x$ and ω_x. We can recover it if it doesn't overlap with the shifted $Z(j\omega)$ on the sides. Therefore, we can write,

$$\omega_c - \omega_z > \omega_x$$
$$\omega_c > \omega_x + \omega_z$$

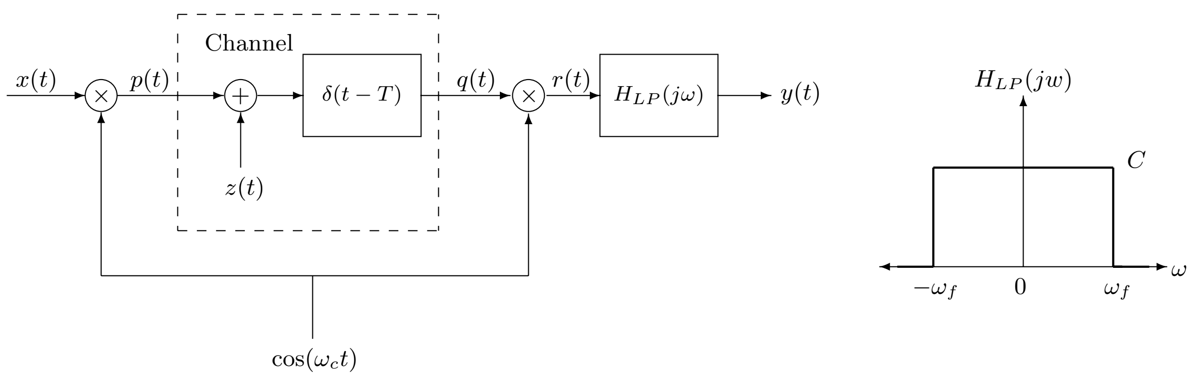
(b) We want to find the parameters C and ω_f for the low-pass filter. From the figure in part (a), we can see that for $\omega \in [-\omega_x, \omega_x]$, $R(j\omega) = \frac{1}{2}X(j\omega)$, so to get back $X(j\omega)$,

$$C = 2.$$

The cutoff frequency ω_f just needs be between the baseband copy of $X(j\omega)$ and the shifted $Z(j\omega)$. Therefore,

$$\omega_x \leq \omega_f \leq \omega_c - \omega_z$$

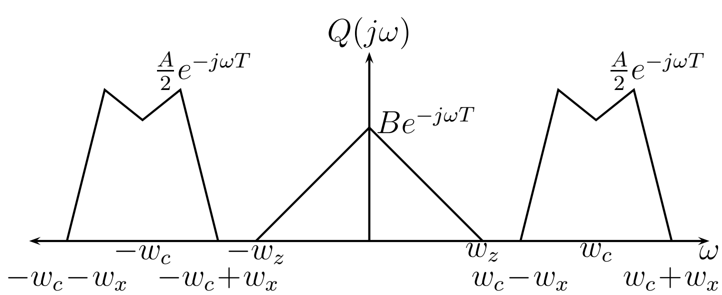
(c) This is the same system as above, except that we need to account for the delay in the channel. We can account for this delay with $h_d(t) = \delta(t-T)$. Following is the system diagram with the delay element.



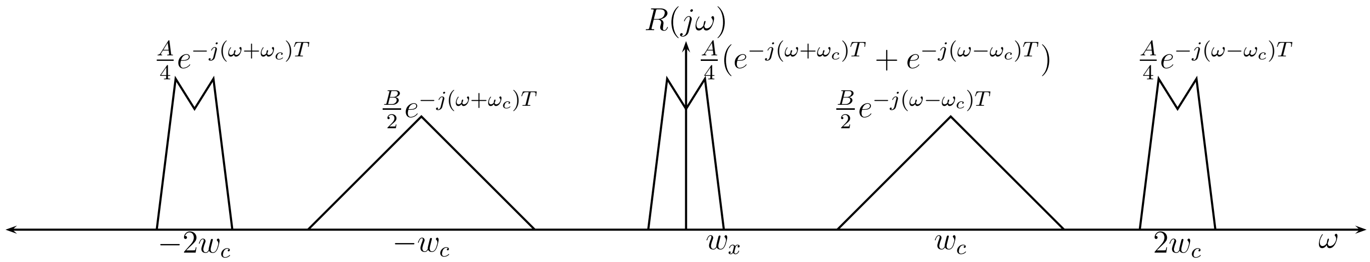
Let us define $s(t)$ to be the signal right before the delay. Then,

$$\begin{aligned} S(j\omega) &= P(j\omega) + Z(j\omega) \\ Q(j\omega) &= S(j\omega)H_d(j\omega) = e^{-j\omega T}S(j\omega). \end{aligned}$$

We are essentially repeating what we did in (a) and (b), but this time $Q(j\omega)$ is scaled by $e^{-j\omega T}$.



Carrying out the the convolution, we see that the new $R(j\omega)$ is:



As we use the parameters chosen in parts (a) and (b), only the baseband (centered around origin) component will pass through the filter. It will be the same as in part (b), except that we use the new "height" factor from the graph. If we let $Y_0(j\omega)$ be the output of the system in part (b), then

$$\begin{aligned} Y(j\omega) &= Y_0(j\omega)\frac{1}{2}(e^{-j(\omega+\omega_c)T}+e^{-j(\omega-\omega_c)T}) \\ &= Y_0(j\omega)e^{-j\omega T}\frac{1}{2}(e^{-j\omega_c T}+e^{j\omega_c T}) \\ &= Y_0(j\omega)e^{-j\omega T}\cos(\omega_c T) \end{aligned}$$

Recalling that the system without delay gave an output $Y_0(j\omega) = X(j\omega)$, we can find the new system's frequency response as following:

$$\begin{aligned} Y(jw) &= X(j\omega)e^{-j\omega T}\cos(\omega_c T) \\ H(j\omega) &= \frac{Y(jw)}{X(jw)} = e^{-j\omega T}\cos(\omega_c T). \end{aligned}$$