MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Fall 2003

PROBLEM SET 7

Issued: October 28, 2003 Due: November 5, 2003

REMINDER: Computer Lab 2 is also due on November 7.

Reading Assignments:

Lectures #14-15 & PS#7: Chapter 7 (through Section 7.4) and Chapter 8 (through Section 8.4) of O&W

Lectures #16-18 & PS#8: Section 7.5 and Chapters 8 and 9 (through Section 9.6) of O&W

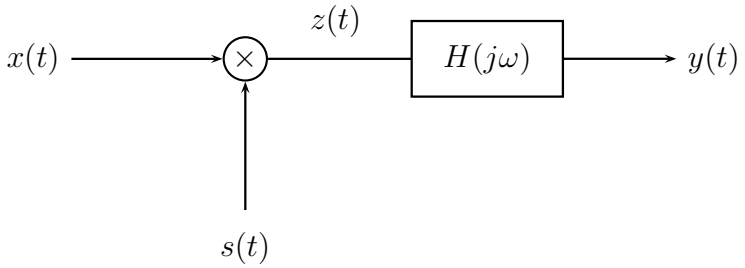
Exercise for home study (not to be turned in, although we will provide solutions):

O&W 7.28

O&W 8.23

Problems to be turned in:

Problem 1 A sinusoidal input signal, $x(t) = \cos(10t)$ is sampled and filtered as shown below.

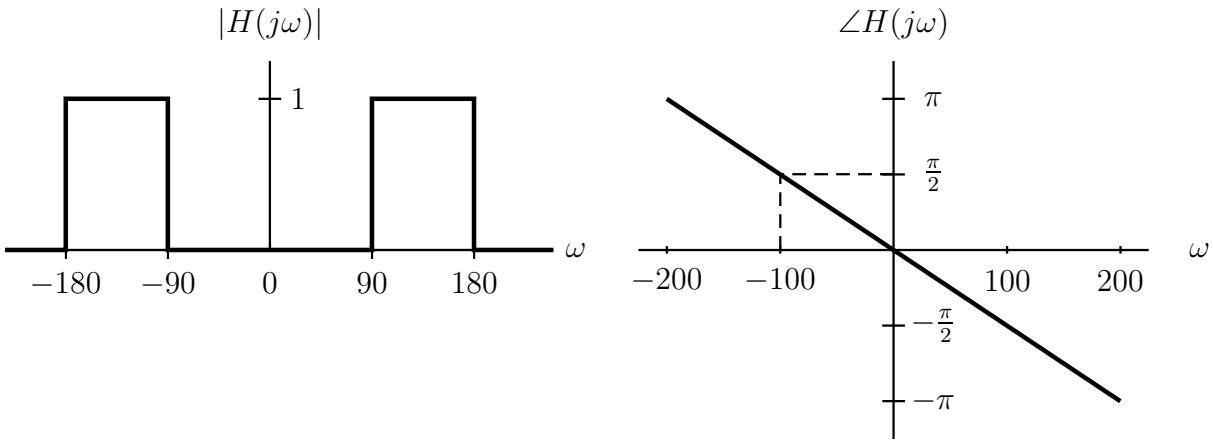


The frequency response of the filter is

$$|H(j\omega)| = \begin{cases} 1, & 90 < |\omega| < 180 \\ 0, & \text{otherwise} \end{cases}$$

$$\angle H(j\omega) = -\frac{\pi\omega}{200},$$

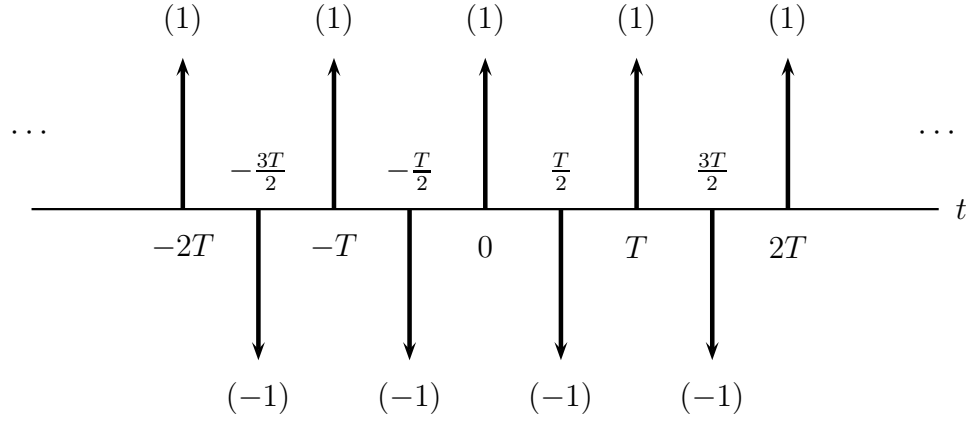
as shown in the figure below:



(a) Suppose $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$ and $T = \frac{2\pi}{90}$. Provide a labeled sketch of $Z(j\omega)$, the Fourier transform of $z(t)$.

(b) Find $y(t)$, assuming the $s(t)$ and the value of T given in part (a).

(c) The sampling function $s(t)$ is changed to the form below, with $T = \frac{2\pi}{90}$.



Find $y(t)$.

Problem 2 O&W 7.30 except let the input be $x_c(t) = \delta(t - T/2)$.

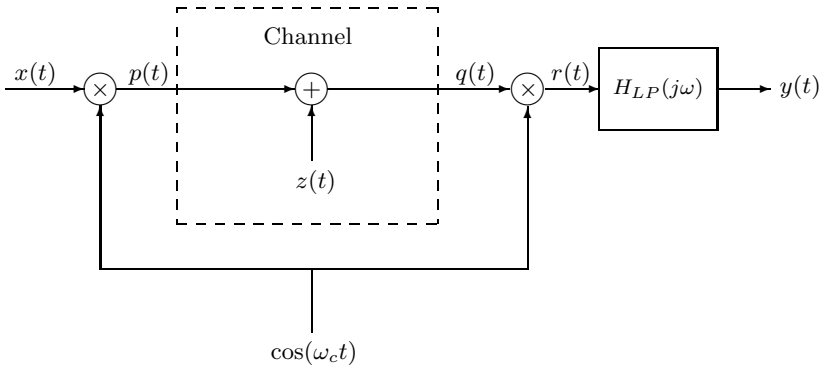
Problem 3 Answer the same questions in O&W 7.31 except let $h[n]$ be the unit sample response of an LTI, causal system with difference equation

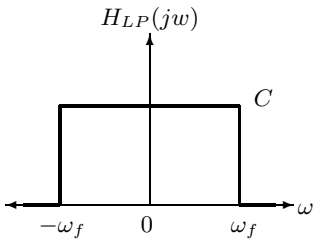
$$y[n] = \frac{3}{4}y[n-2] + x[n] + \frac{1}{4}x[n-1].$$

Problem 4 O&W 8.22

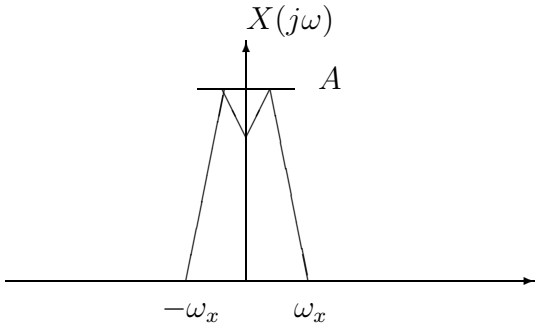
Problem 5 O&W 8.49

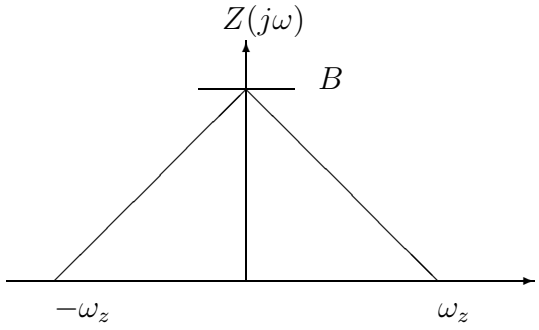
Problem 6 The transmission system depicted below is intended to allow a signal $x(t)$ to be transmitted through a communication channel that also carries other signals represented by $z(t)$.





Both $x(t)$ and $z(t)$ are bandlimited, and their Fourier transforms $X(j\omega)$ and $Z(j\omega)$ are real, as sketched below. Notice that the bandwidth ω_z of $Z(j\omega)$ is much greater than the bandwidth ω_x of $X(j\omega)$.



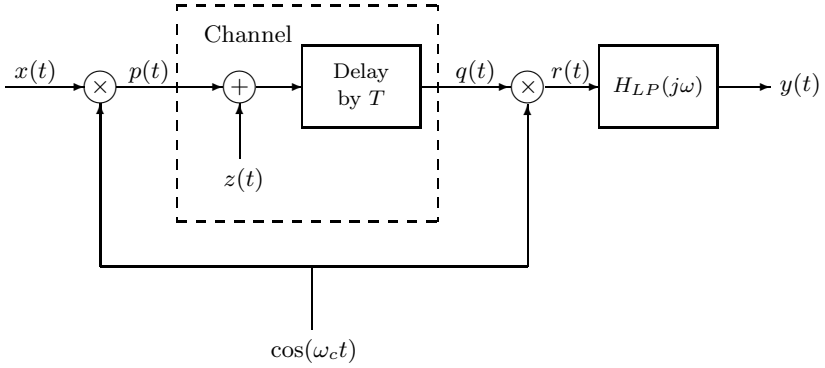


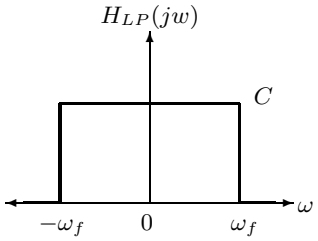
(a) We wish to determine parameters for the transmission system so that the output $y(t)$ is equal to the input $x(t)$. Determine the range of ω_c for which $y(t)$ can be made equal to $x(t)$. Explain.

(b) Given a value of ω_c in the range specified in part (a), determine the range of values of ω_f and the value of C for which $y(t) = x(t)$. Your expression may contain ω_c and/or parameters of the Fourier transforms $X(j\omega)$ and $Z(j\omega)$. Briefly explain your reasoning.

(c) Consider next what would happen if the channel also had appreciable delay, as depicted below. Assuming that the parameters are chosen as in parts (a) and (b), find an expression for the system's frequency response, defined as

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$





Reminder: The first 20 problems in each chapter of O&W have answers included at the end of the text. Consider using these for additional practice, either now or as you study for tests.