MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Fall 2003

Quiz 1
Tuesday, October 14, 2003

Directions: The exam consists of 5 problems on pages 2 to 19 and work space on pages 20 and 21. Please make sure you have all the pages. Tables of Fourier series properties are supplied to you at the end of this booklet. **Enter all your work and your answers directly in the spaces provided on the printed pages of this booklet. Please make sure your name is on all sheets. DO IT NOW!**. All sketches must be adequately labeled. Unless indicated otherwise, **answers must be derived or explained**, not just simply written down. This examination is closed book, but students may use one 8 1/2 $\times$ 11 sheet of paper for reference. Calculators may not be used.

NAME:____________________________________

Check your section	Section	Time	Rec. Instr.
☐	1	10-11	Prof. Zue
☐	2	11-12	Prof. Zue
☐	3	1- 2	Prof. Gray
☐	4	11-12	Dr. Rohrs
☐	5	12- 1	Prof. Voldman
☐	6	12- 1	Prof. Gray
☐	7	10-11	Dr. Rohrs
☐	8	11-12	Prof. Voldman

Please leave the rest of this page blank for use by the graders:

Problem	No. of points	Score	Grader
1	18		
2	20		
3	20		
4	21		
5	21		
Total	100		

PROBLEM 1 (18%)

For the questions in this problem, no explanation is necessary.

Consider the following three systems:

SYSTEM A: $y(t) = x(t+2)\sin(\omega t + 2),$ where $\omega \neq 0$

SYSTEM B: $y[n] = \left(-\frac{1}{2}\right)^n (x[n]+1)$

SYSTEM C: $y[n] = \sum_{k=1}^{n} \left(x^2[k+1] - x[k]\right)$

where x and y are the input and output of each system.

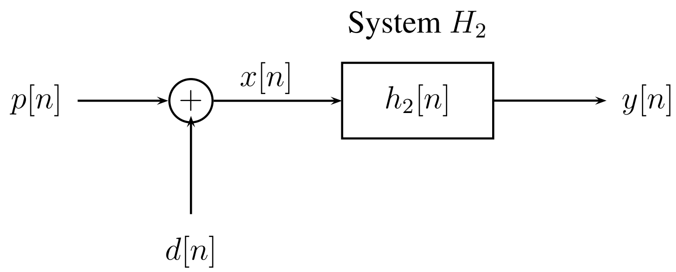
Circle YES or NO for each of the following questions for each of these three systems.

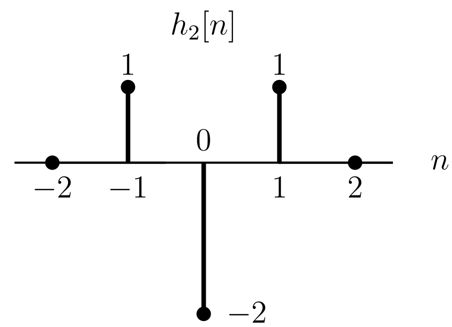
	SYSTEM A	SYSTEM B	SYSTEM C
Is the system linear ?	YES NO	YES NO	YES NO
Is the system time invariant ?	YES NO	YES NO	YES NO
Is the system causal ?	YES NO	YES NO	YES NO
Is the system stable ?	YES NO	YES NO	YES NO

Work Page for Problem 1

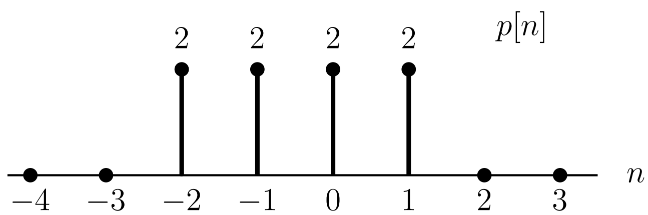
PROBLEM 2 (20%)

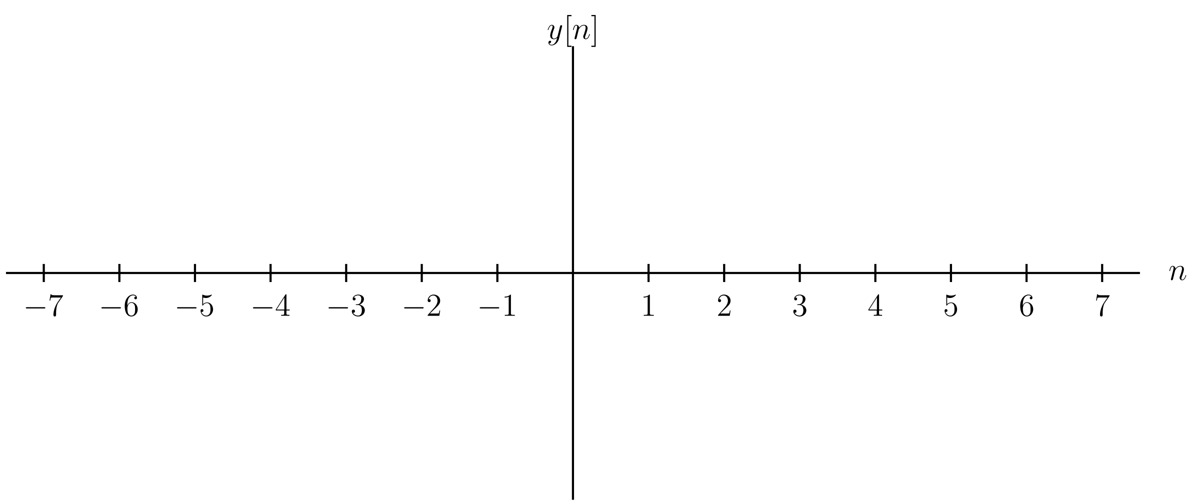
Consider a DT LTI system, H_2 with a unit sample response $h_2[n] = h[n] * h[n+1]$, as shown below, where $h[n] = \delta[n] - \delta[n-1]$. You may remember from one of the lectures that $h[n]$ can be viewed as the unit sample response of a DT LTI system that acts as an edge detector. The purpose of this problem is to develop an edge detector that is robust against additive noise.





Part a. Assume that the input to the system, $p[n]$ is as shown below, and there is no noise, i.e., $d[n] = 0$ and $p[n] = x[n]$. Provide a labeled sketch of $y[n]$, the output of the system.





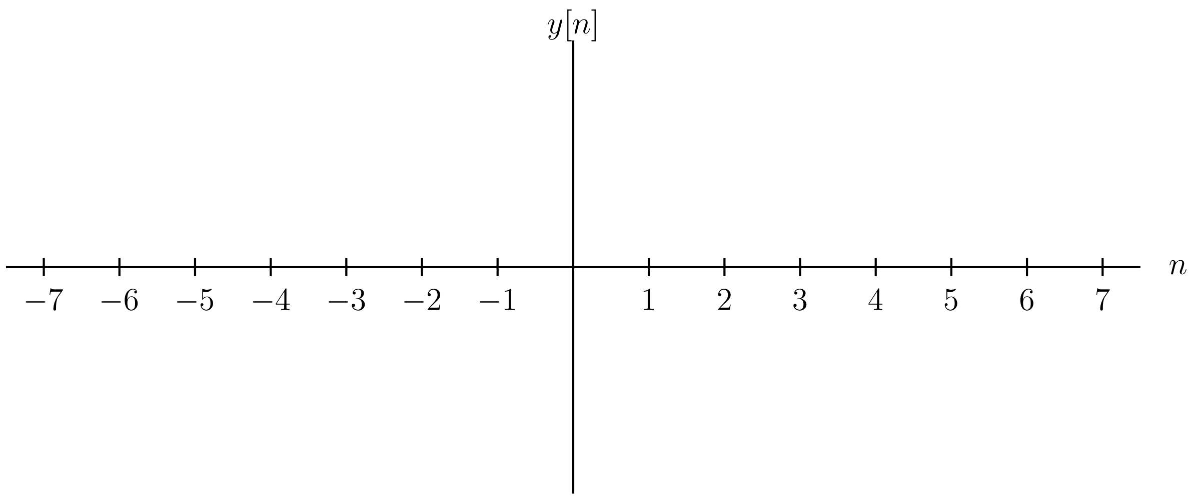
Work Page for Problem 2

 Problem 2 continues on the following page.

Part b. For the same input signal as **Part a.**, now assume that the noise signal is

$$d[n] = -\delta[n+1].$$

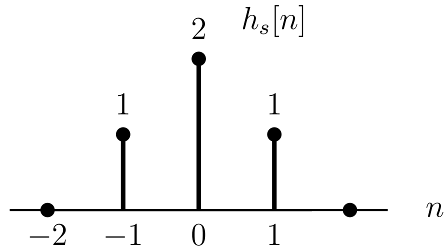
Provide a labeled sketch of the output $y[n]$, i.e., the response to $x[n] = p[n] + d[n]$.

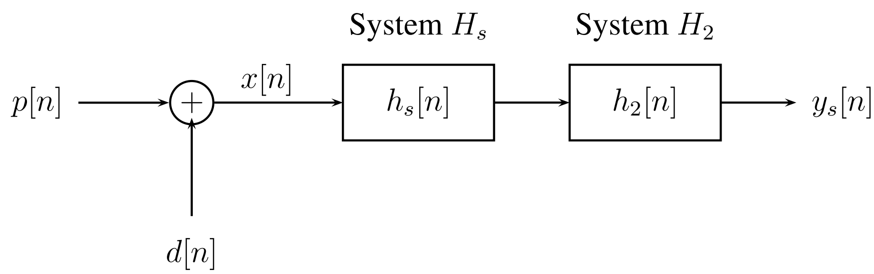


Work Page for Problem 2

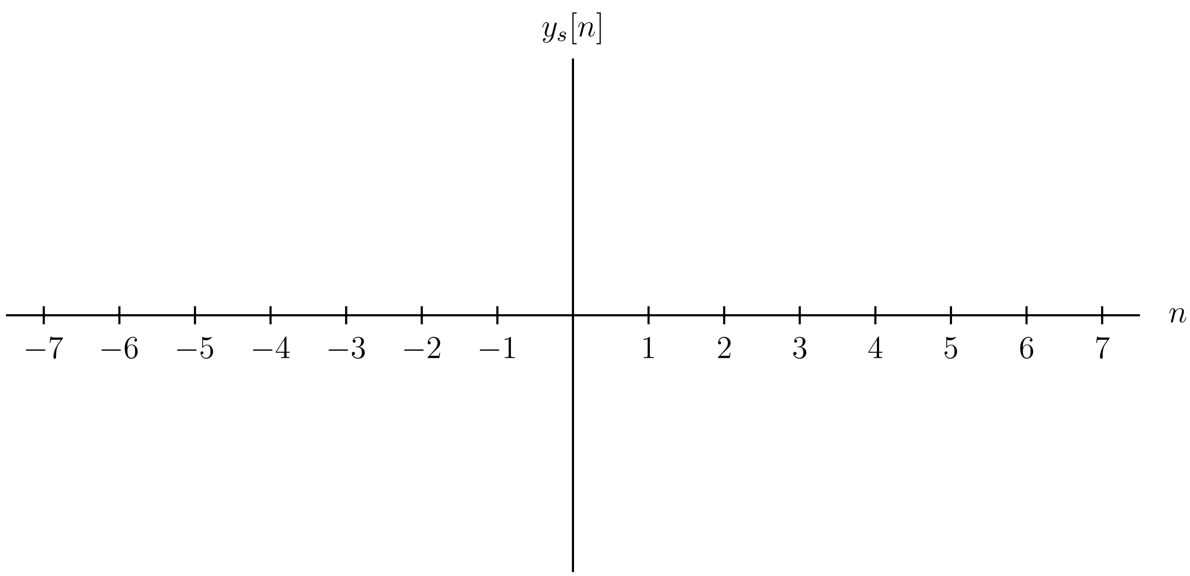
Problem 2 continues on the following page.

Part c. In order to use system H_2 as a part of an edge detector, we would like to add an LTI system H_s whose unit sample response, $h_s[n]$ is shown below. System H_s smoothes out effect of noise on $x[n]$. The overall system can be represented as below:





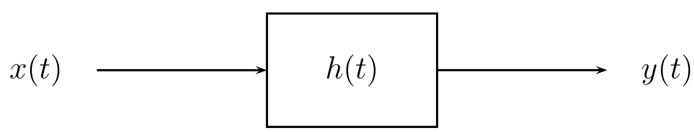
Provide a labeled sketch of the overall output $y_s[n]$, when $p[n]$ and $d[n]$ are exactly the same as in **Part b.**

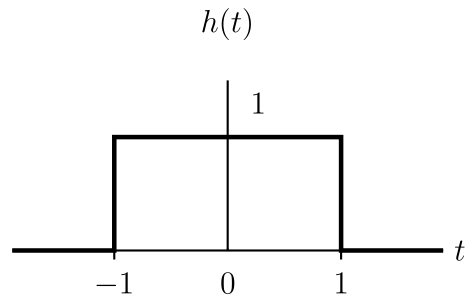


Work Page for Problem 2

PROBLEM 3 (20%)

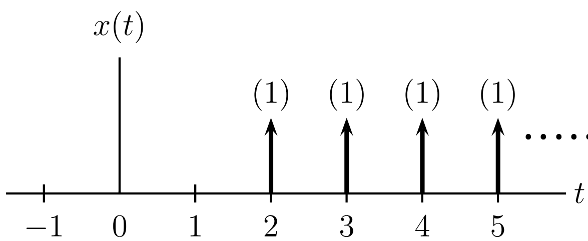
Consider the CT LTI system whose impulse response is given as:



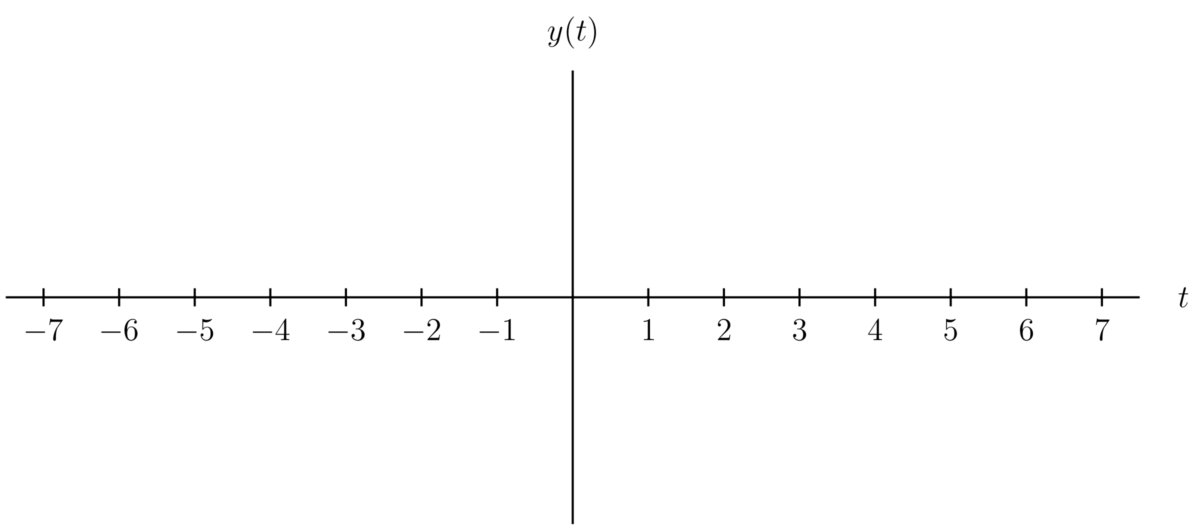


The following two parts can be done independently.

Part a. The input $x(t)$, an impulse train starting at $t = 2$, is depicted below:



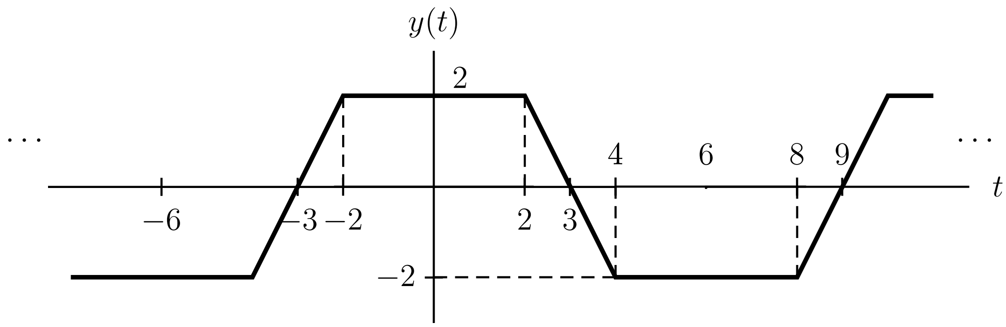
Provide a labeled sketch of the corresponding output $y(t)$.



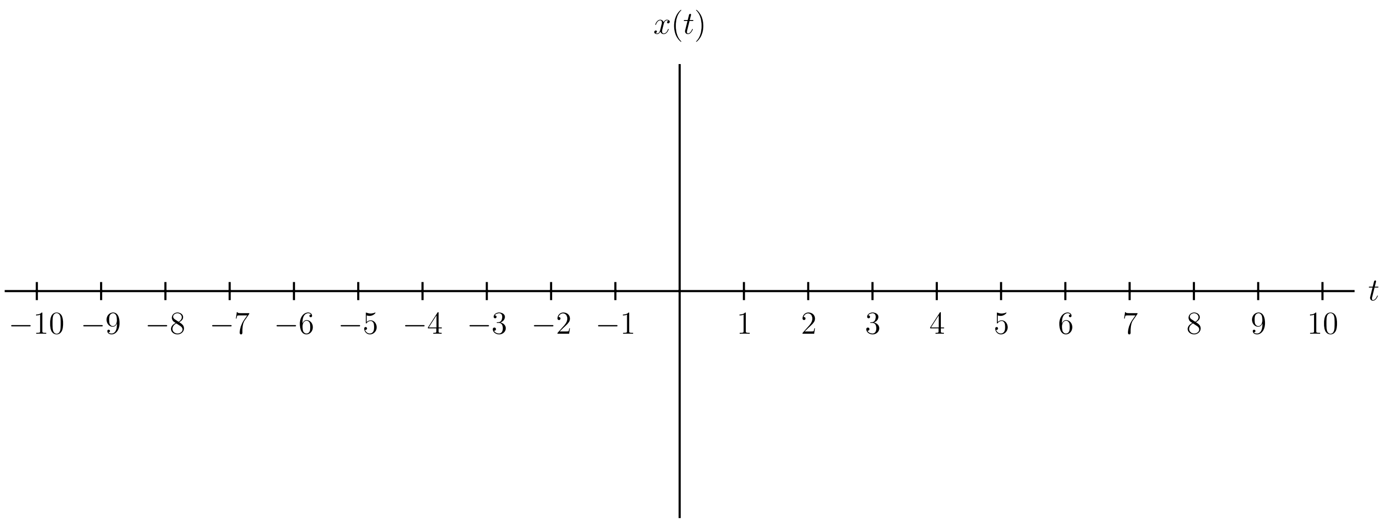
Work Space for Problem 3

 Problem 3 continues on the following page.

Part b. For this part, the output $y(t)$ is periodic and is depicted below:



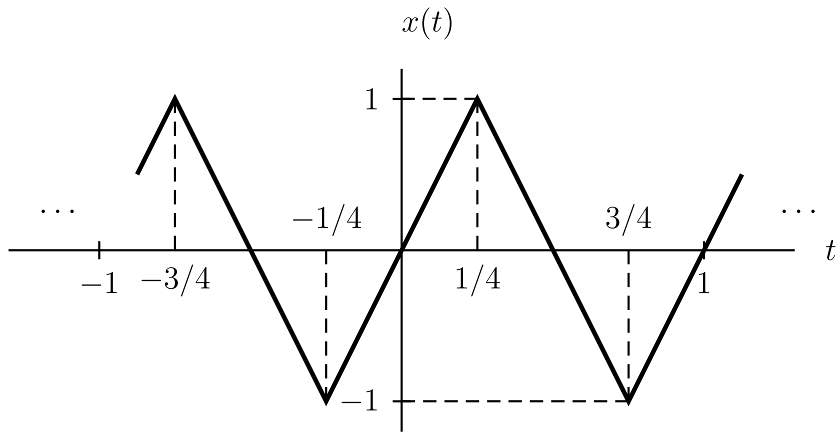
Provide a labeled sketch of the input $x(t)$ that produces this $y(t)$.



Work Page for Problem 3

PROBLEM 4 (21%)

Consider the following periodic triangular wave shown below:



Part a. Determine the Fourier series coefficients, a_k for $x(t)$.

$a_k =$

Work Page for Problem 4

 Problem 4 continues on the following page.

Part b. Consider a causal LTI system, S, whose input-output relation is characterized by the following stable linear constant coefficient differential equation:

$$\frac{d^2y}{dt^2} + 4\pi\frac{dy}{dt} + 4\pi^2 y(t) = 4\pi^2 x(t),$$

where $x(t)$ is the input and $y(t)$ is the output of the system. Suppose $x(t)$ shown on the previous page is applied to the system S as an input. Let b_k be the Fourier coefficients of the corresponding output $y(t)$. Find b_3 and b_{-3}.

$b_3 =$ ____________________ $b_{-3} =$ ____________________.

NAME:____________________________________

Work Page for Problem 4

PROBLEM 5 (21%)

You are given the following facts about a discrete time sequence $x[n]$:

(a) $x[n]$ is real and odd.

(b) $x[n]$ is periodic with period $N = 6$.

(c) $\frac{1}{N} \sum_{n=<N>} |x[n]|^2 = 10$.

(d) $\sum_{n=<N>} (-1)^{n/3} x[n] = 6j$.

(e) $x[1] > 0$.

Find an expression of $x[n]$ in the form of sines and cosines.

$x[n] =$ ____

Work Page for Problem 5

NAME:___________________________________

Work Page

NAME:____________________________________

Work Page