MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Fall 2003

Quiz 2
Thursday, November 13, 2003

<u>Directions:</u> The exam consists of 6 problems on pages 2 to 19 and work space on pages 20 and 21. Please make sure you have all the pages. Tables of Fourier series properties as well as CT Fourier transform and DT Fourier transform properties and tables are supplied to you at the end of this booklet. **Enter all your work and your answers directly in the spaces provided on the printed pages of this booklet. Please make sure your name is on all sheets. You may use bluebooks for scratch work, but we will not grade them at all.** All sketches must be adequately labeled. Unless indicated otherwise, **answers must be derived or explained**, not just simply written down. This examination is closed book, but students may use two 8 1/2 $\times$ 11 sheets of paper for reference. Calculators may not be used.

NAME:____________________________________

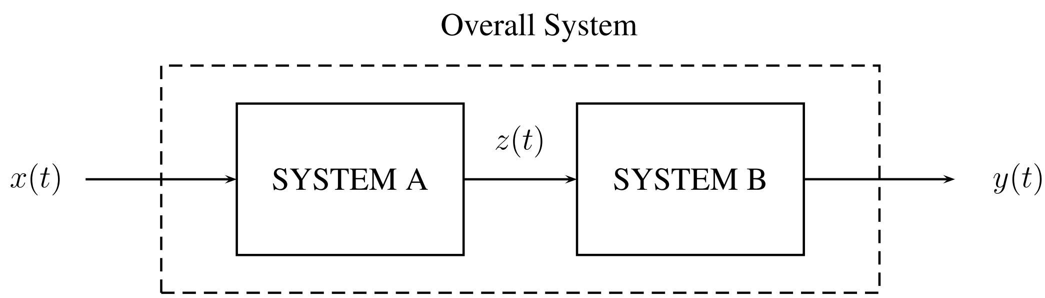
Check your section	Section	Time	Rec. Instr.
☐	1	10-11	Prof. Zue
☐	2	11-12	Prof. Zue
☐	3	1- 2	Prof. Gray
☐	4	11-12	Dr. Rohrs
☐	5	12- 1	Prof. Voldman
☐	6	12- 1	Prof. Gray
☐	7	10-11	Dr. Rohrs
☐	8	11-12	Prof. Voldman

Please leave the rest of this page blank for use by the graders:

Problem	No. of points	Score	Grader
1	15		
2	20		
3	25		
4	25		
5	15		
Total	100		

PROBLEM 1 (15%)

Consider the following system depicted below:



The input-output relation for SYSTEM A is characterized by the following causal LCCDE:

$$\frac{dz(t)}{dt} + 6z(t) = \frac{dx(t)}{dt} + 5x(t),$$

and the impulse response $h_b(t)$ for SYSTEM B is defined as:

$$h_b(t) = e^{-10t}u(t).$$

Part a. What is the frequency response of the complete system ? That is, given $x(t) \stackrel{\mathcal{F}}{\longleftrightarrow} X(j\omega)$ and $y(t) \stackrel{\mathcal{F}}{\longleftrightarrow} Y(j\omega)$, determine $H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)}$.

$H(j\omega) =$ ______________________________

Work Page for Problem 1

 Problem 1 continues on the following page.

Part b. What is the impulse response, $h(t)$ of the complete system ?

$h(t) =$ ____

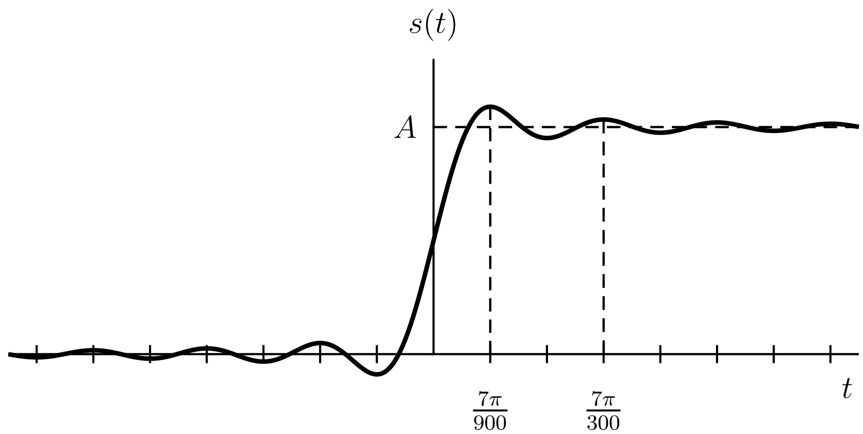
Part c. What is the differential equation that relates $x(t)$ and $y(t)$?

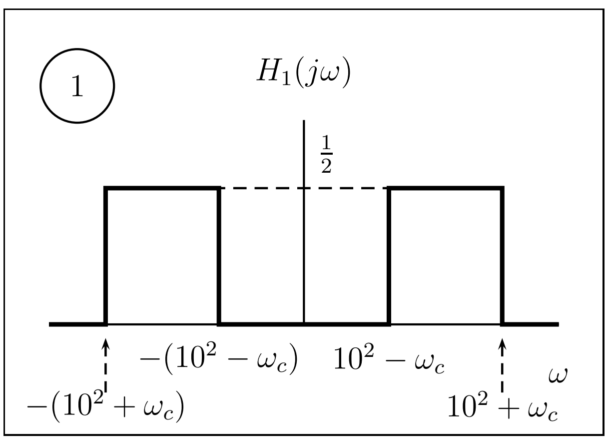
____

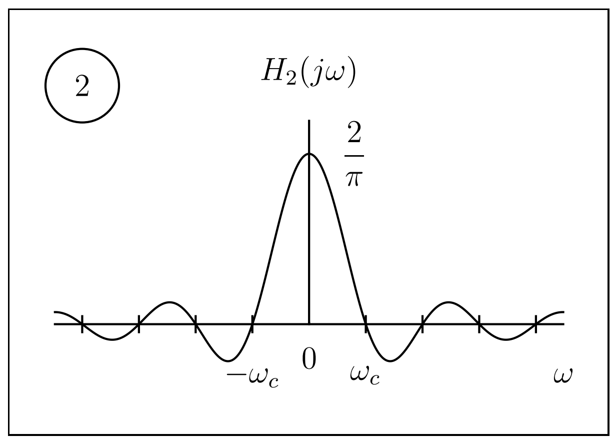
Work Page for Problem 1

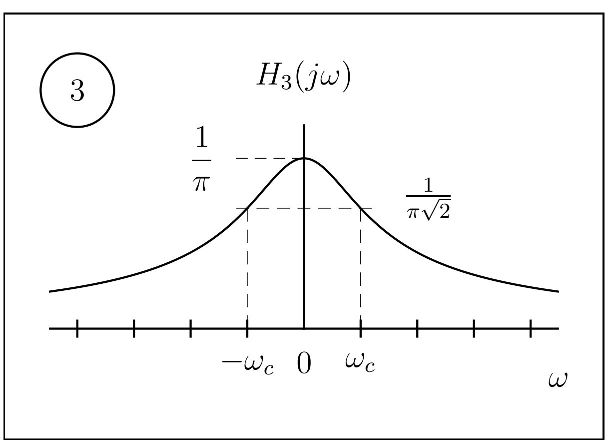
PROBLEM 2 (20%)

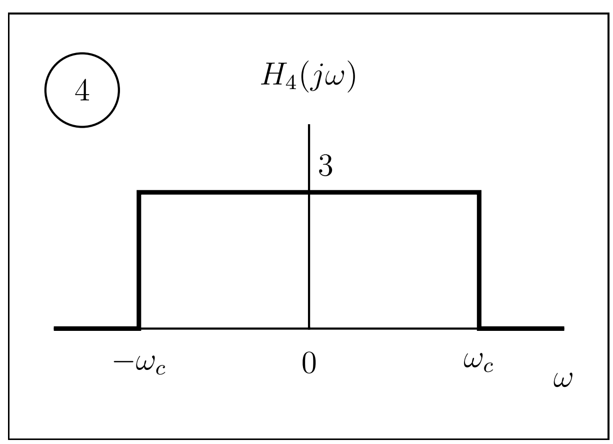
Part a. Match the **step response** $s(t)$ below to the correct frequency response and give a brief justification to your answer in the space provided in the next page.











SYSTEM ______________

Brief justification (You can show why your answer is correct or show why the other three systems are not correct) :

 Problem 2 continues on the following page.

Part b. Find ω_c and A.

$\omega_c =$ ______________________ $A =$ ______________________

Work Page for Problem 2

PROBLEM 3 (25%)

Part a. Determine the Fourier transform $R(e^{j\omega})$ of the following sequence:

$$r[n] = \begin{cases} 1, & 0 \le n \le M,\ M \text{ is a positive even integer} \\ 0, & \text{otherwise.} \end{cases}$$

$R(e^{j\omega}) =$ ________________________________

Part b. Consider the sequence

$$w[n] = \begin{cases} \frac{1}{2}\left(1 - \cos\left(\frac{2\pi n}{M}\right)\right), & 0 \le n \le M \\ 0, & \text{otherwise,} \end{cases}$$

where M is as defined in **Part a.** Express $W(e^{j\omega})$, the Fourier transform of $w[n]$ in terms of $R(e^{j\omega})$, the Fourier transform of $r[n]$ above.

$W(e^{j\omega}) =$ ________________________________

Work Page for Problem 3

 Problem 3 continues on the following page.

Part c. Is there a positive even integer M that will make $W(e^{j\omega})$ real ? If so, find the values of M that satisfy this constraint. If not, explain why.

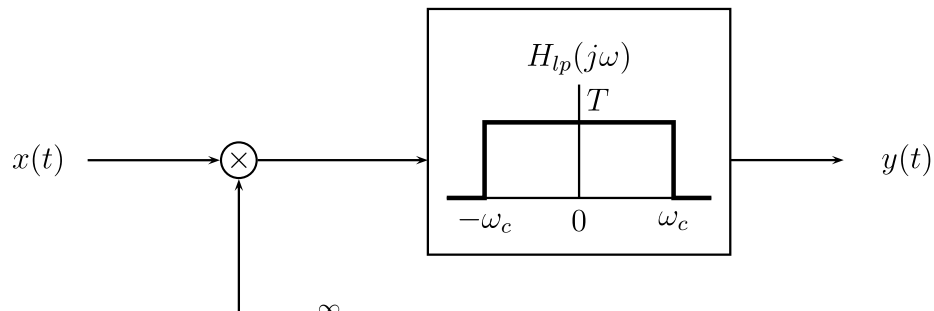
YES	**NO**
Values of M________________________	Explanation:

Work Page for Problem 3

PROBLEM 4 (25%)

Part a is independent of the other parts in this problem.

Part a. Consider the following system:



$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT), \quad \omega_s = \frac{2\pi}{T}$$

For this part, suppose

$$x(t) = \left(\frac{\sin(4\pi t)}{\pi t}\right)\left(\frac{\sin(2\pi t)}{\pi t}(-1)^t\right),$$

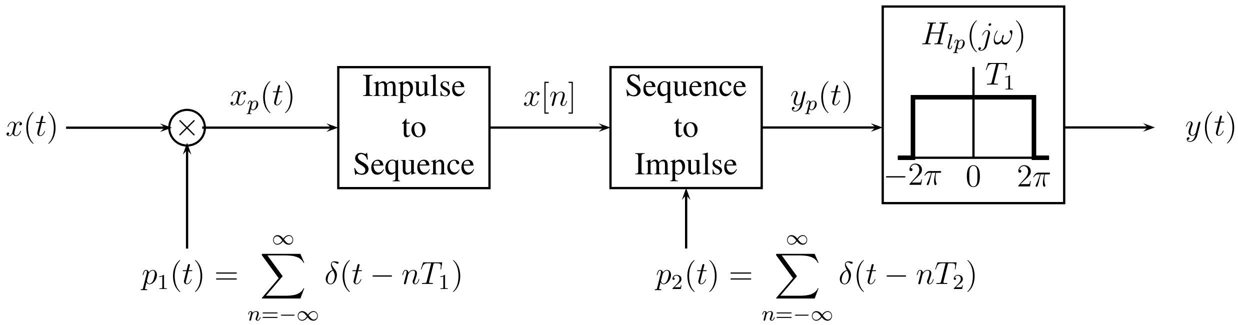
and $p(t)$ is an impulse train of frequency ω_s. $H_{lp}(j\omega)$ is a lowpass filter whose gain is T and cutoff frequency is ω_c. Determine the cutoff frequency ω_c and a frequency ω_0 such that $y(t) = x(t)$ for any $\omega_s > \omega_0$.

$\omega_0 =$ ____________________ , $\omega_c =$ ____________________

Work Page for Problem 4

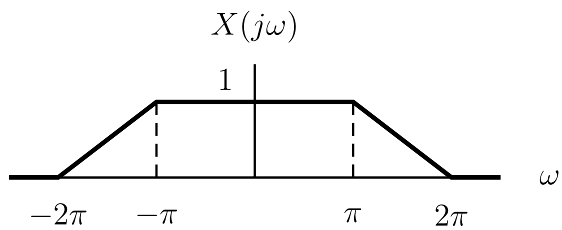
 Problem 4 continues on the following page.

For the rest of this problem, let's consider the following system:



$p_1(t)$ is an impulse train whose fundamental period is T_1 and $p_2(t)$ is another impulse train whose fundamental period is T_2. $H_{lp}(j\omega)$ is a lowpass filter whose gain is T_1 and cutoff frequency is at ω_c. Note that $x[n] = x(nT_1)$ and $y_p(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT_2)$.

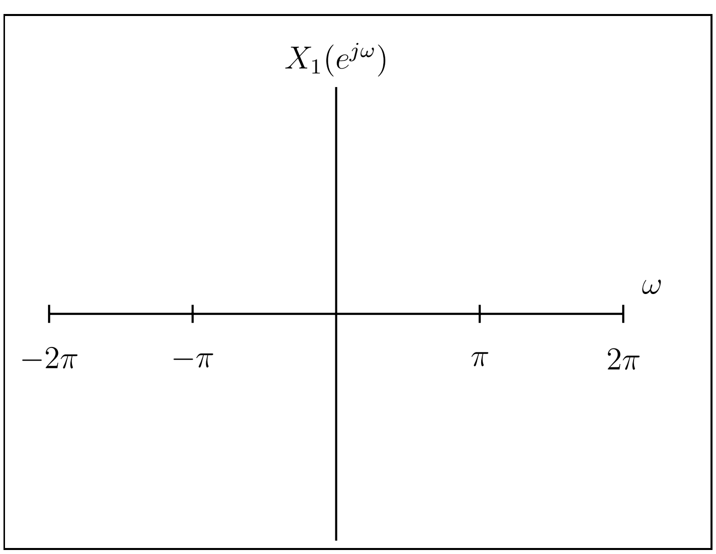
The input $x(t)$ is a band limited real signal whose Fourier transform is shown below:

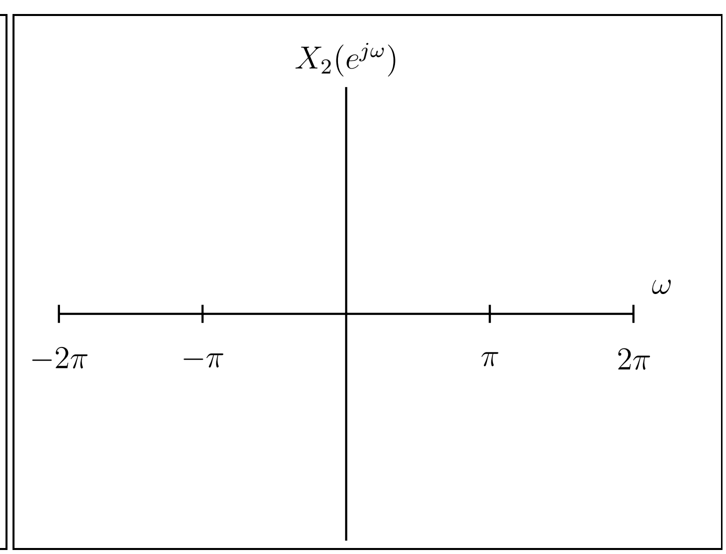


Part b. Let's define

$$x_1[n] = x(nT_1), \quad \text{where } T_1 = 1,$$
$$x_2[n] = x(nT_1), \quad \text{where } T_1 = \frac{1}{3}.$$

In the given axes below and on the top of the next page, provide the labeled sketches of $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$, Fourier transforms of $x_1[n]$ and $x_2[n]$ respectively.

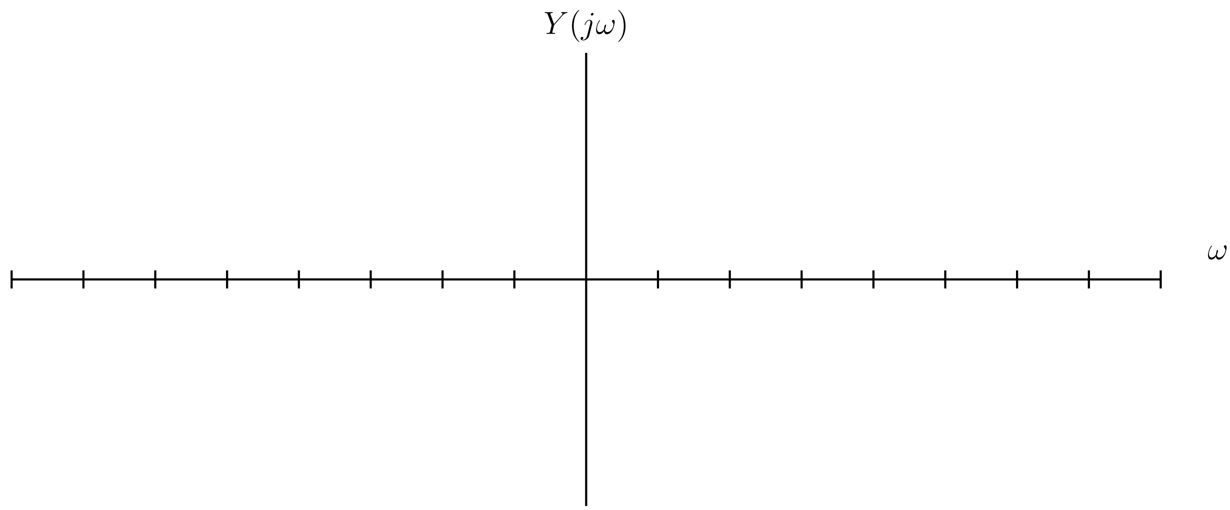




Work Page for Problem 4

 Problem 4 continues on the following page.

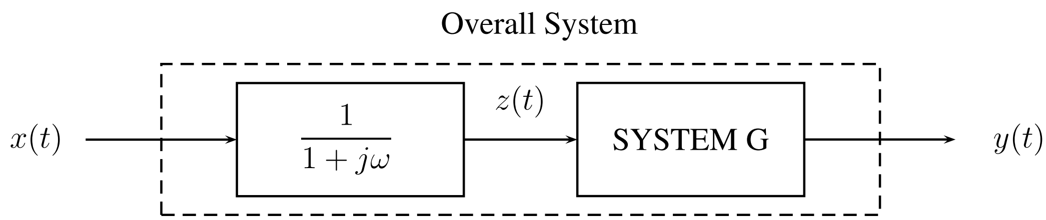
Part c. Suppose $T_1 = \frac{1}{3}$ and $T_2 = \frac{1}{2}$. Provide a labeled sketch of $Y(j\omega)$, Fourier transform of the overall output $y(t)$.



Work Page for Problem 4

PROBLEM 5 (15%)

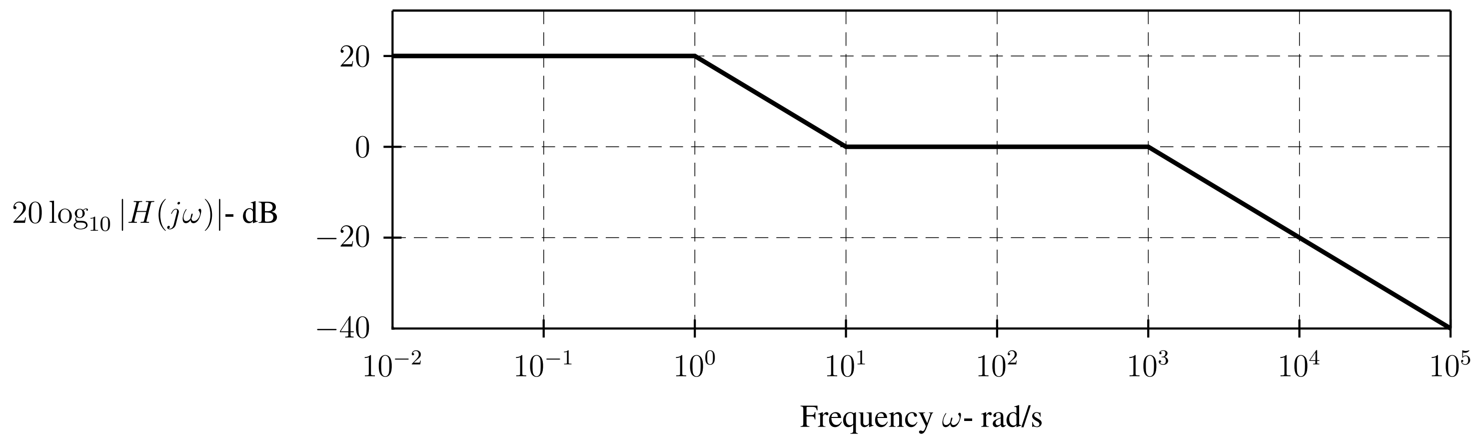
We have a cascade of two stable CT LTI systems as shown below:

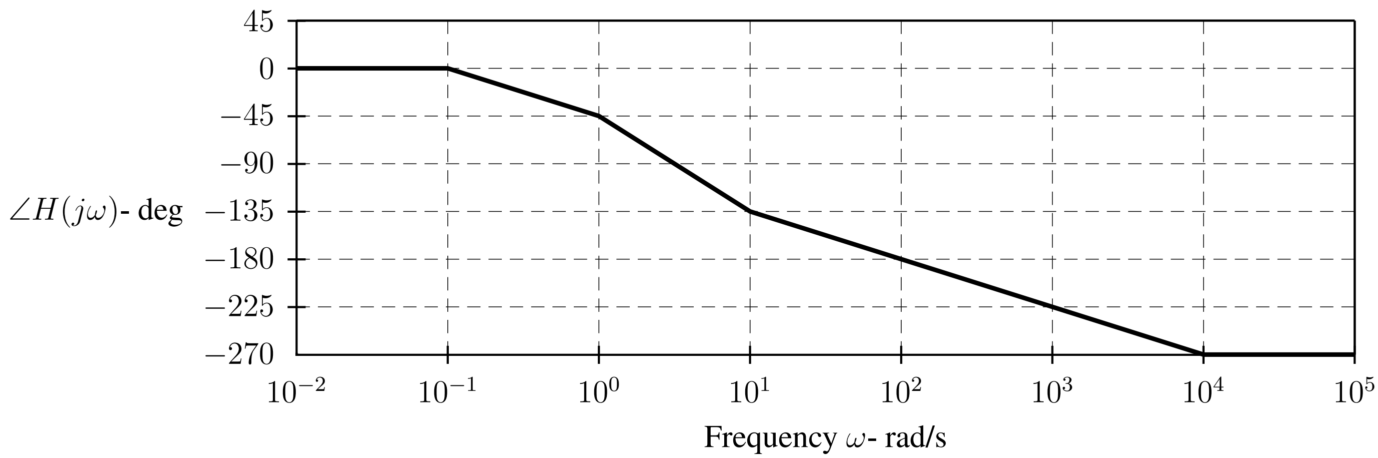


The straight line approximation of Bodé plots of the overall system, $H(j\omega)$ is shown in the next page.

Find the frequency response, $G(j\omega)$, of SYSTEM G.

$G(j\omega)$ = ____


20
0
$20\log_{10}|H(j\omega)|$- dB
−20
−40
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
Frequency ω- rad/s


45
0
−45
−90
$\angle H(j\omega)$- deg
−135
−180
−225
−270
10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2} 10^{3} 10^{4} 10^{5}
Frequency ω- rad/s

Work Page

Work Page