Signals and Systems

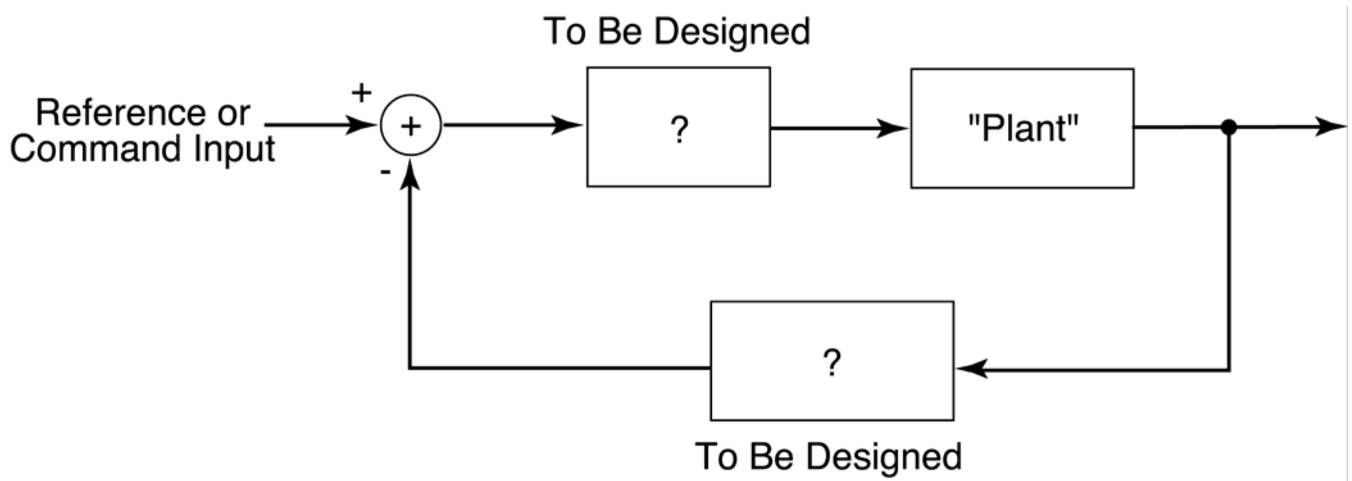
Fall 2003

Lecture #20

20 November 2003

1. Feedback Systems
2. Applications of Feedback Systems

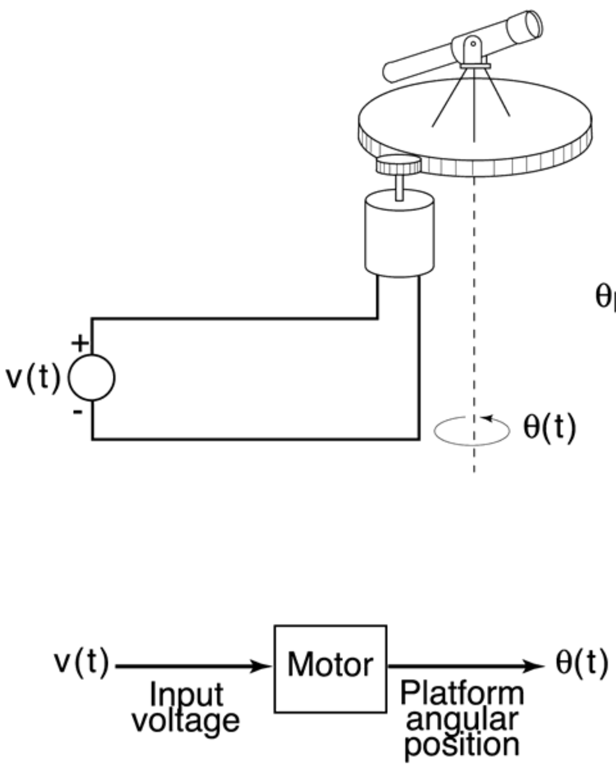
A Typical Feedback System

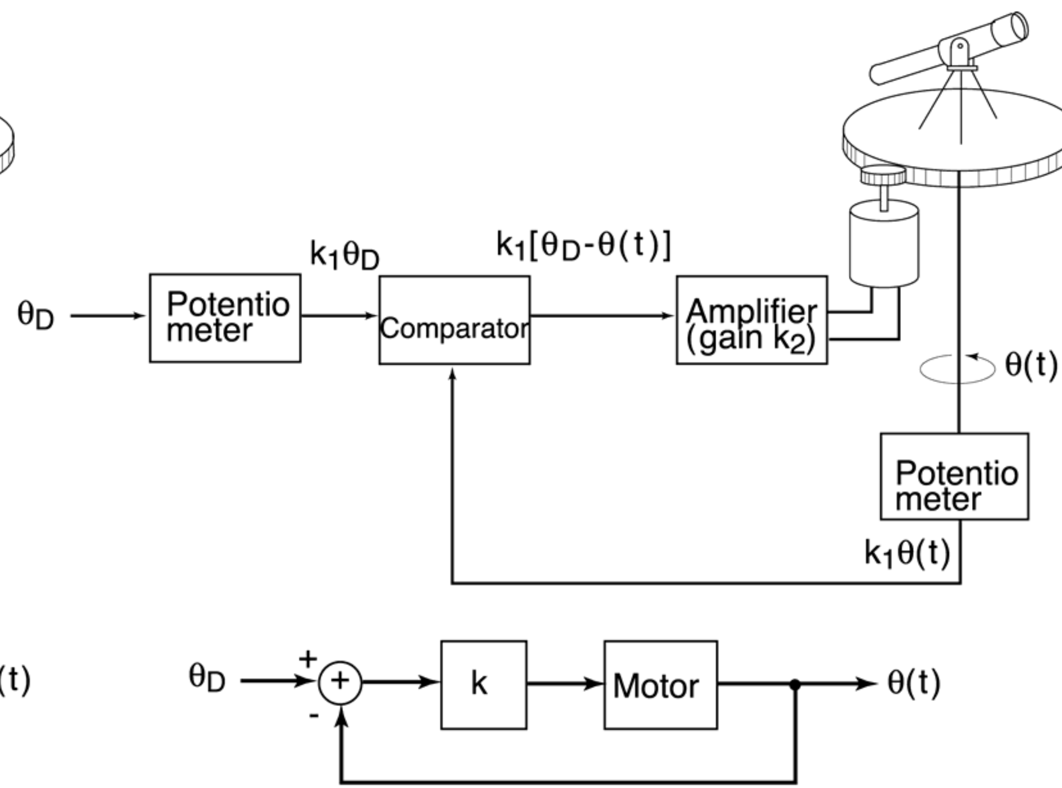


Why use Feedback?

- Reducing Effects of Nonidealities
- Reducing Sensitivity to Uncertainties and Variability
- Stabilizing Unstable Systems
- Reducing Effects of Disturbances
- Tracking
- Shaping System Response Characteristics (bandwidth/speed)

One Motivating Example



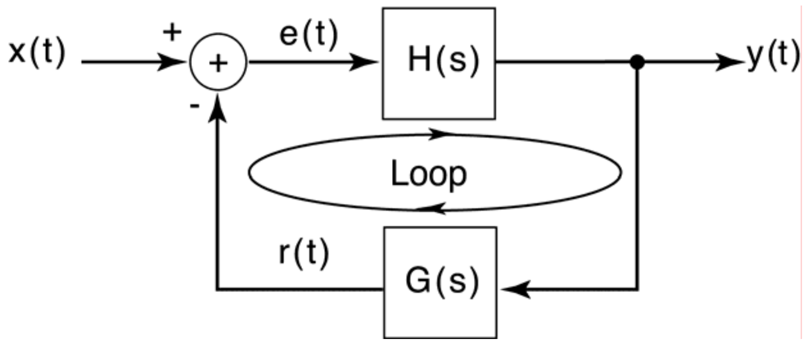


Open-Loop System

Closed-Loop Feedback System

Analysis of (Causal!) LTI Feedback Systems: Black's Formula

CT System



$$Q(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)H(s)}$$

Black's formula (1920's)

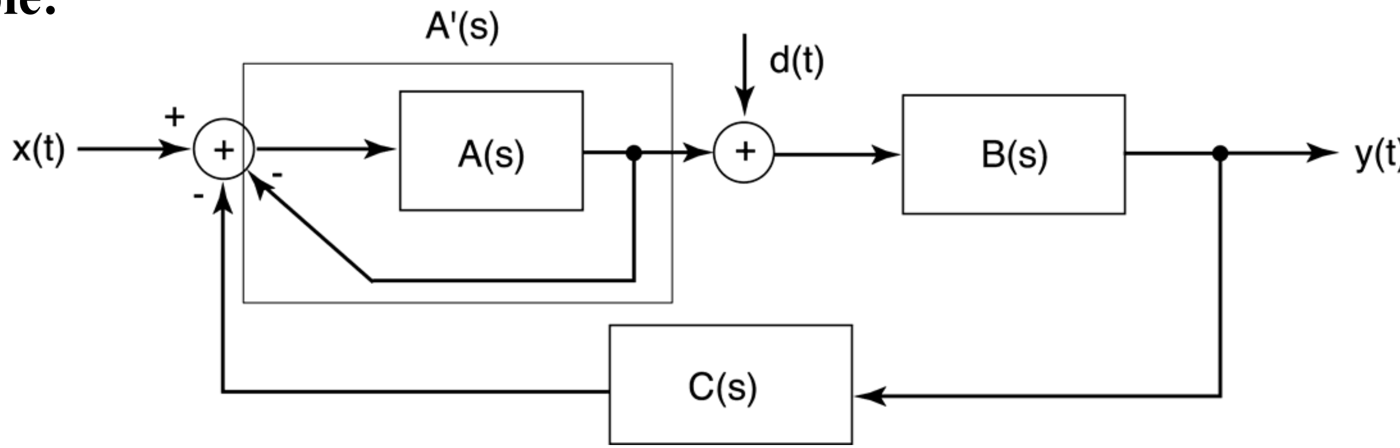
$$\text{Closed - loop system function } = \frac{\text{forward gain}}{1 \ - \ \text{loop gain}}$$

Forward gain — total gain along the forward path from the input to the output

Loop gain — total gain around the closed loop

Applications of Black's Formula

Example:

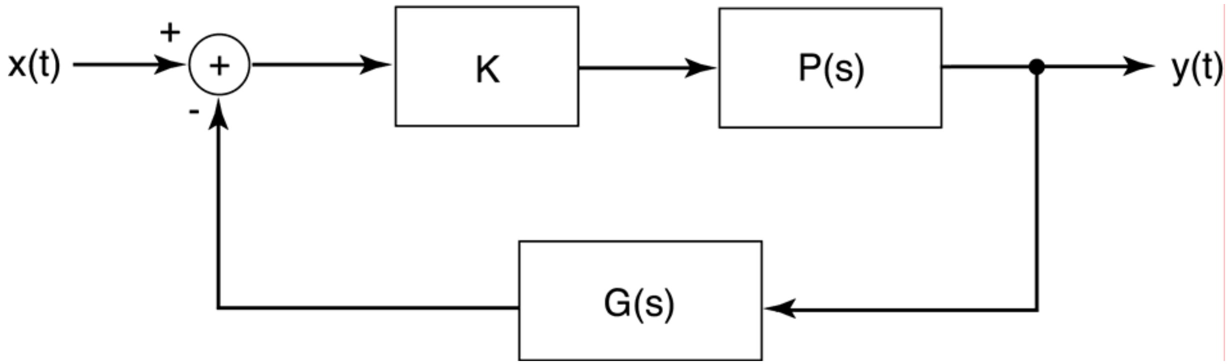


1) $$\frac{Y(s)}{X(s)} = \frac{\text{Forward gain}}{1-\text{loop gain}} = \frac{A'B}{1+A'BC}$$

$$A' = \frac{A}{1+A} \Rightarrow \frac{Y(s)}{X(s)} = \frac{AB}{1+A+ABC}$$

2) $$\frac{Y(s)}{D(s)} = \frac{\text{Forward gain}}{1-\text{loop gain}} = \frac{B}{1+A'BC} = \frac{B(1+A)}{1+A+ABC}$$

The Use of Feedback to Compensate for Nonidealities



Assume $KP(j\omega)$ is *very large* over the frequency range of interest. In fact, assume

$$|KP(j\omega)G(j\omega)| >> 1$$

$$\Downarrow$$

$$Q(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{KP(j\omega)}{1 + KP(j\omega)G(j\omega)} \approx \frac{1}{G(j\omega)}$$ — Independent of P(s)!!

Example of Reduced Sensitivity

1)The use of operational amplifiers
2)Decreasing amplifier gain sensitivity

Example:

(a) Suppose $KP(j\omega_1) = 1000 \quad , \quad G(j\omega_1) = 0.099$

$$Q(j\omega_1) = \frac{1000}{1 + (1000)(0.099)} = 10$$

(b) Suppose $KP(j\omega_2) = 500 \quad , \quad G(j\omega_2) = 0.099$

(50% gain change)

$$Q(j\omega_2) = \frac{500}{1 + (500)(0.099)} \cong 9.9 \quad \text{(1\% gain change)}$$

Fine, but why doesn't $G(j\omega)$ fluctuate ?

Note:

$$Q(j\omega) \approx \frac{1}{G(j\omega)}$$

$$\Downarrow$$

For amplification, $G(j\omega)$ must *attenuate*, and it is much easier to build attenuators (*e.g.* resistors) with desired characteristics

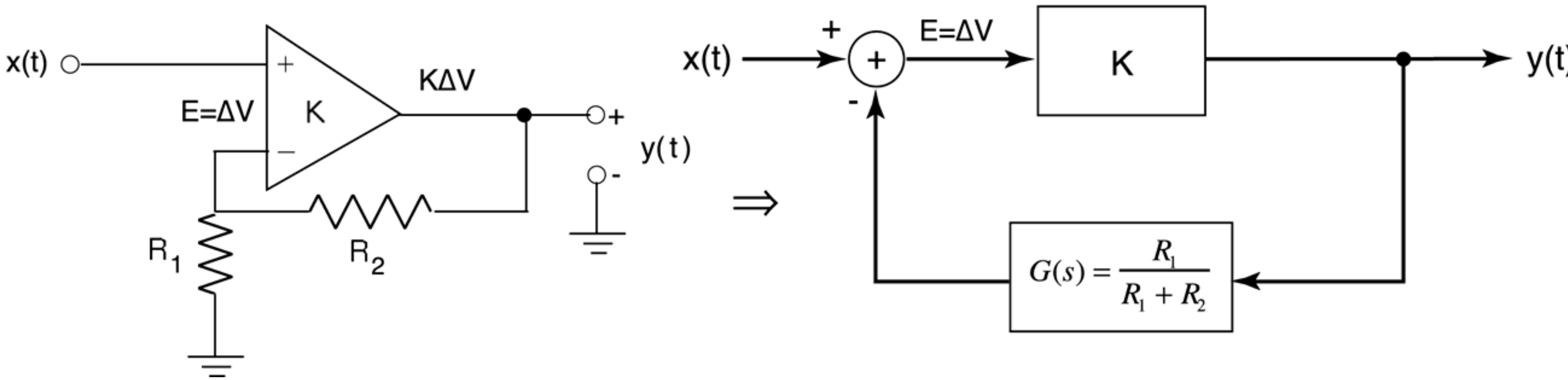
There is a price:

$$|KPG(j\omega)| >> 1 \Rightarrow |KP(j\omega)| >> \frac{1}{|G(j\omega)|}$$

Needs a large loop gain to produce a *steady* (and *linear*) gain for the whole system.

$\Rightarrow$ Consequence of the *negative* (*degenerative*) feedback.

Example: Operational Amplifiers



If the amplitude of the loop gain

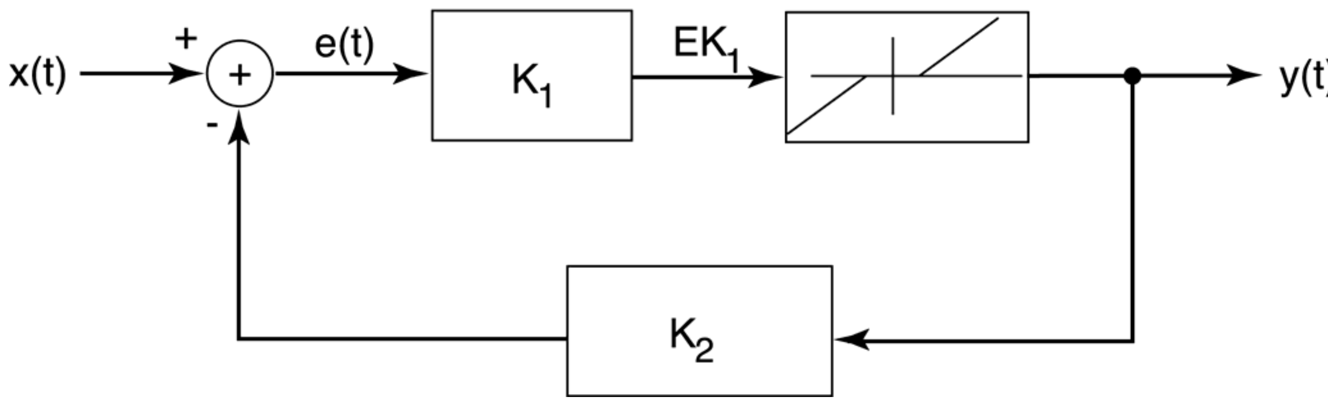
$|KG(s)| >> 1$ — usually the case, unless the battery is totally dead.

Then $$\frac{Y(s)}{X(s)} \approx \frac{1}{G(s)} = \frac{R_1 + R_2}{R_1}$$ —— Steady State

The closed-loop gain only depends on the *passive* components (R_1 & R_2), independent of the gain of the open-loop amplifier K.

The Same Idea Works for the Compensation for Nonlinearities

Example and Demo:
Amplifier with a Deadzone



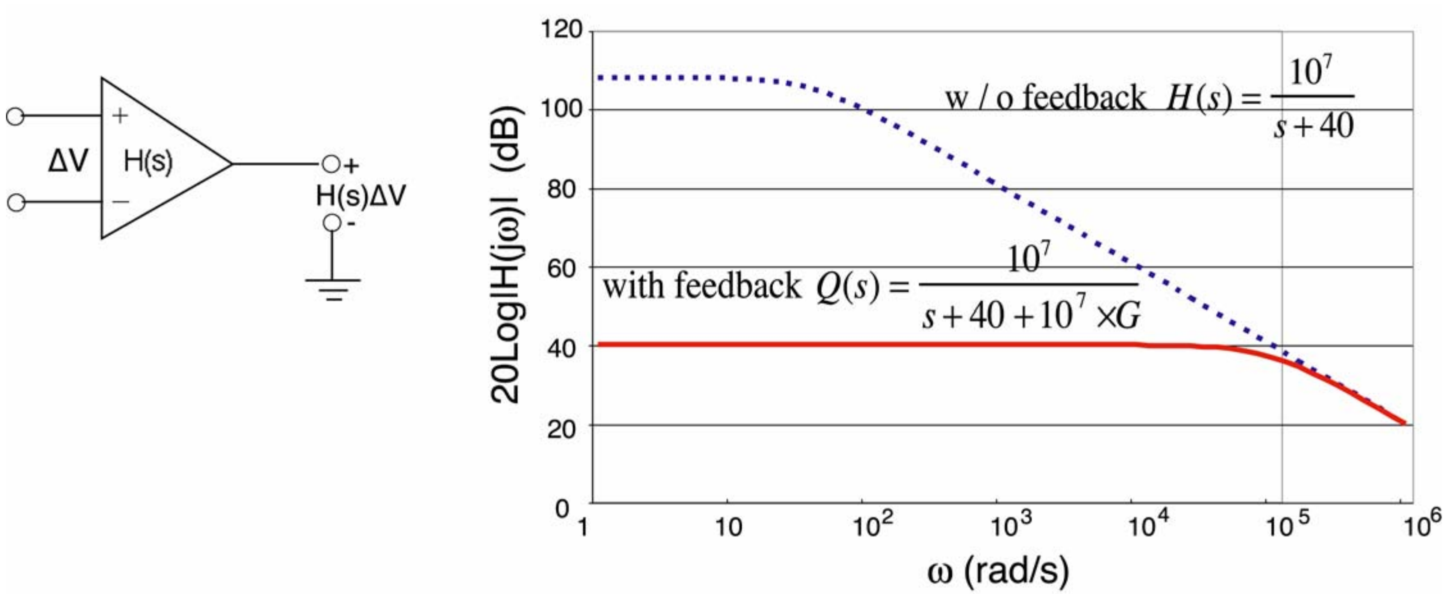
The second system in the forward path has a nonlinear input-output relation (a deadzone for small input), which will cause distortion if it is used as an amplifier. However, as long as the amplitude of the "loop gain" is large enough, the input-output response $\cong 1/K_2$

Improving the Dynamics of Systems

Example: Operational Amplifier 741

The open-loop gain has a very large value at dc but very limited bandwidth

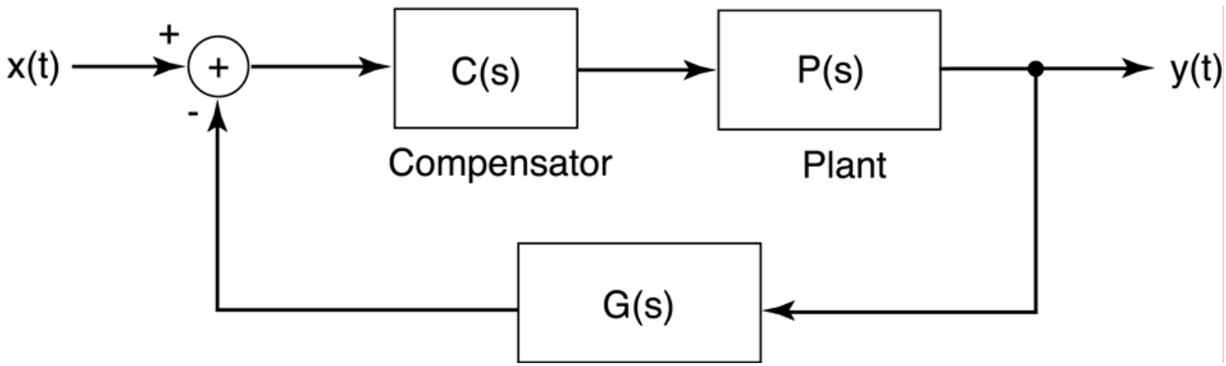
$$H(s) = \frac{10^7}{s+40} \quad \text{—— Not very useful on its own}$$



$$\text{With feedback} \quad Q(s) = \frac{H(s)}{1+G(s)H(s)} = \frac{10^7}{s+40+10^7 G(s)}$$

—— Much broader bandwidth, also $Q(0) \approx 1/G$

Stabilization of Unstable Systems



- $P(s)$ — unstable
- Design $C(s)$, $G(s)$ so that the closed-loop system

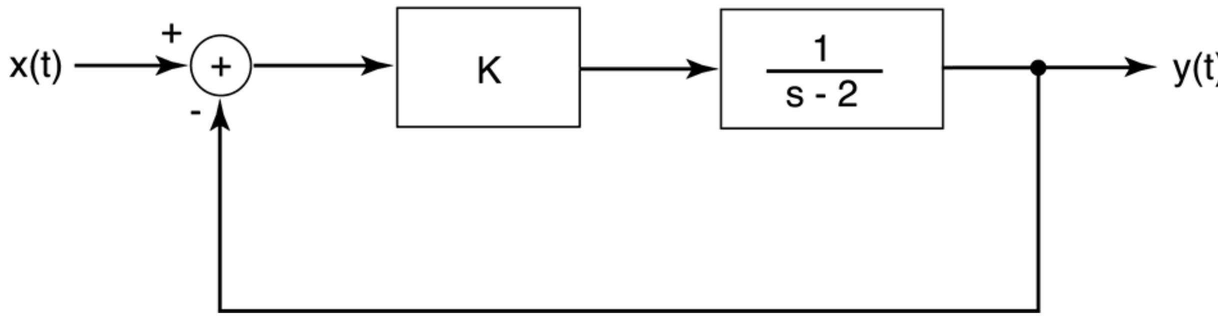
$$Q(s) = \frac{C(s)P(s)}{1 + C(s)P(s)G(s)}$$

is stable

$\Rightarrow$ *poles* of $Q(s)$ = *roots* of $1+C(s)P(s)G(s)$ in LHP

Example #1: First-order unstable systems

$$P(s) = \frac{1}{s-2}$$
$$G(s) = 1$$



Try: $C(s) = K$ proportional feedback

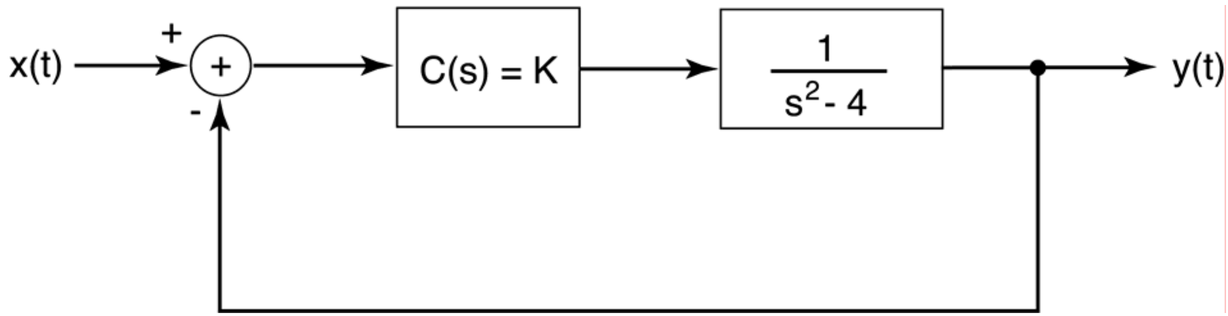
$$Q(s) = \frac{\frac{K}{s-2}}{1+\frac{K}{s-2}} = \frac{K}{s-2+K}$$

Stable as long as $K > 2$

Example #2: Second-order unstable systems

$$P(s) = \frac{1}{s^2 - 4}$$

$$G(s) = 1$$



Attempt #1: Proportional Feedback $C(s) = K$

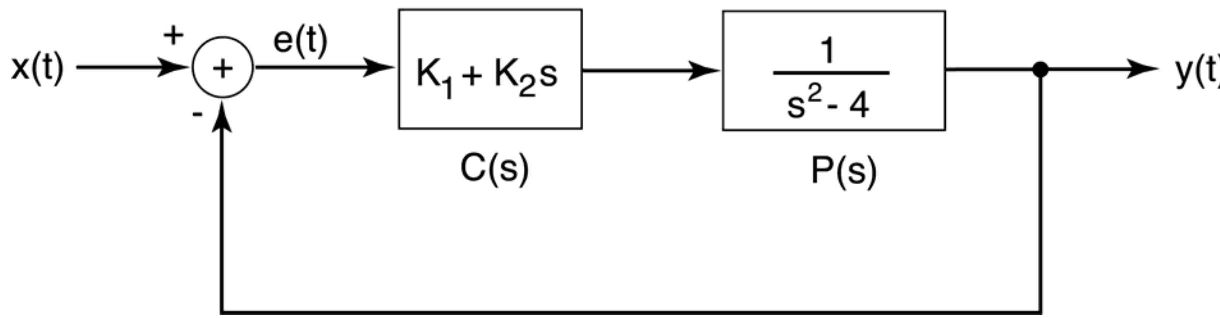
$$Q(s) = \frac{\frac{K}{s^2-4}}{1 + \frac{K}{s^2-4}} = \frac{K}{s^2 - 4 + K}$$

— Unstable for *all* values of K

— Physically, need damping — a term proportional to $s \Leftrightarrow d/dt$

Example #2 (continued):

Attempt #2: Try Proportional-Plus-Derivative (PD) Feedback



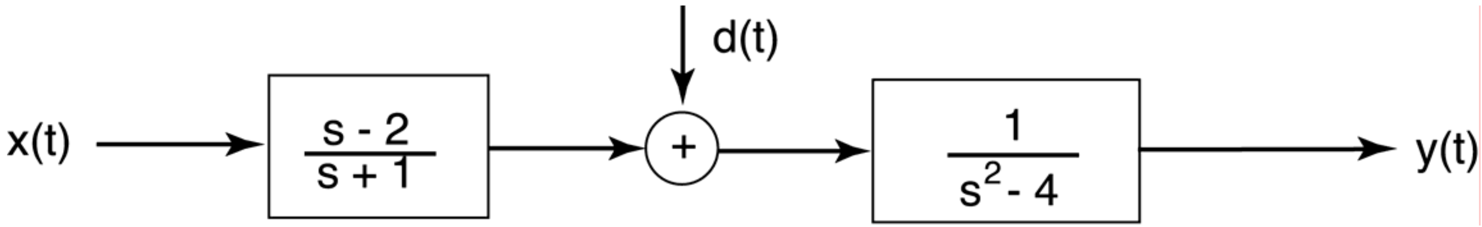
$$C(s) = K_1 + K_2 s$$

$$\begin{aligned} Q(s) &= \frac{\frac{K_1 + K_2 s}{s^2 - 4}}{1 + \frac{K_1 + K_2 s}{s^2 - 4}} \\ &= \frac{K_1 + K_2 s}{s^2 + K_2 s + (K_1 - 4)} \end{aligned}$$

— Stable as long as $K_2 > 0$ (sufficient damping) and $K_1 > 4$ (sufficient gain).

Example #2 (one more time):

Why didn't we stabilize by canceling the unstable poles?

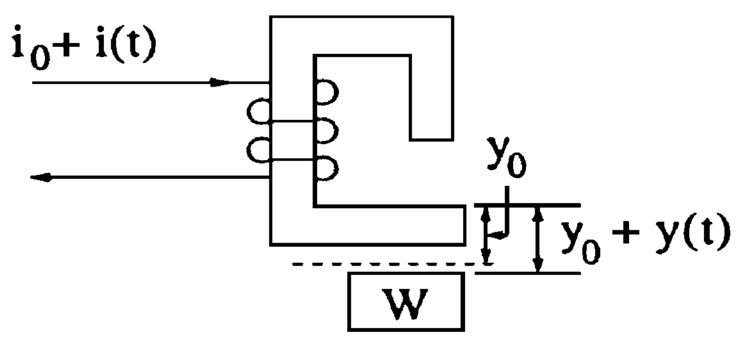


There are at least *two* reasons why this is a really bad idea:

a) In real physical systems, we can *never* know the precise values of the poles, it could be $2 \pm \Delta$.

b) Disturbance between the two systems will cause instability.

Demo: Magnetic Levitation



i_o = current needed to balance the weight W at the rest height y_o

Force balance $$\frac{W}{g}\frac{d^2y}{dt^2} = W - \frac{(i_0 + i(t))^2}{(y_0 + y(t))}$$

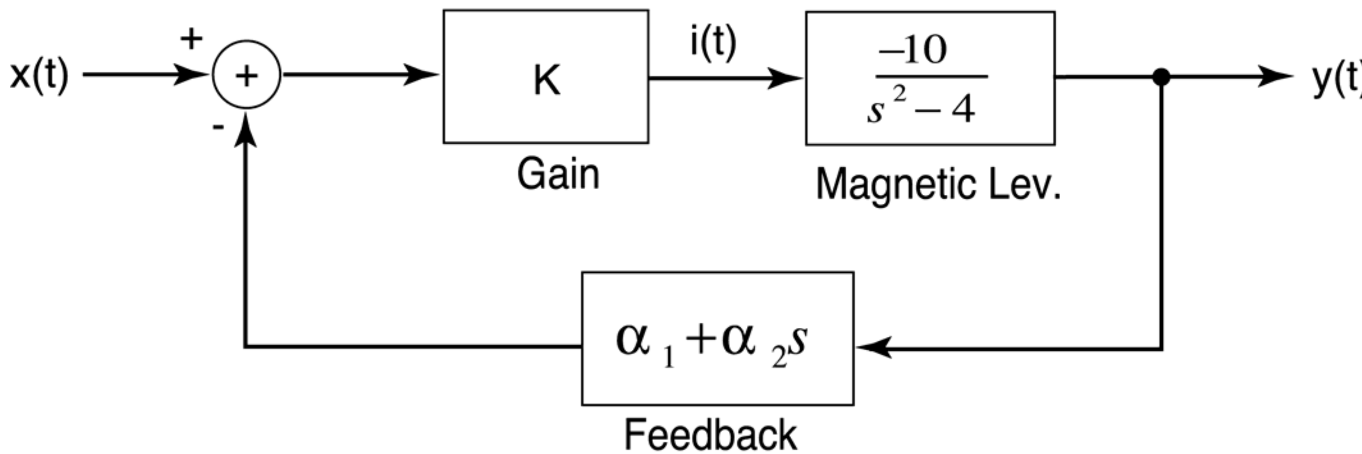
Linearize about equilibrium with specific values for parameters

$$\frac{dy^2}{dt^2} = 4y(t) - 10i(t)$$

$$\Downarrow$$

$$Y(s) = \left(\frac{-10}{s^2 - 4}\right) I(s)$$ — Second-order unstable system

Magnetic Levitation (Continued):



$$Q(s) = \frac{-10K}{s^2 - 10K\alpha_2 s - (4 + 10K\alpha_1)}$$

$$E.g.: \quad K = 1, \quad \alpha_1 = -1.3, \quad \alpha_2 = -0.6$$

$$\Downarrow$$

$$Q(s) = \frac{-10}{(s+3)^2}$$ — Stable!