MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.003: Signals and Systems—Fall 2003

PROBLEM SET 10

Issued: November 25, 2003 Due: December 5, 2003

REMINDER: Computer Lab 3 is also due on December 5.

Reading Assignments:

Lectures #21-22 & PS#10: Chapters 10 & 11 (through Subsection 11.3.4) of O&W

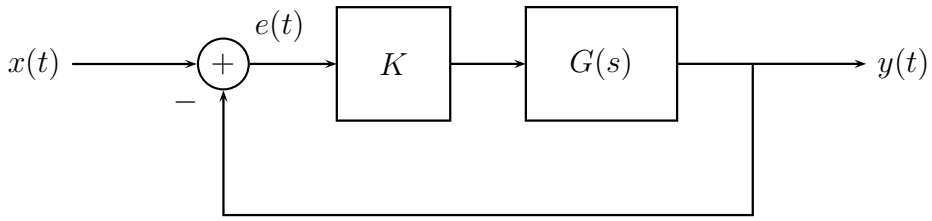
Lectures #23-24 & PS#11: Chapters 10 & 11 (through Subsection 11.3.4) of O&W

Exercise for home study (not to be turned in, although we will provide solutions):

(E1) O&W 11.32 (a) through (d)

Problems to be turned in:

Problem 1 Consider the following feedback configuration.



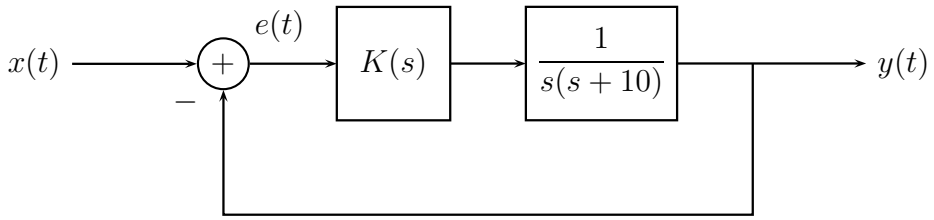
Sketch the root loci for $K > 0$ and $K < 0$ for each of the following:

(a) $G(s) = \dfrac{1}{s+1}$.

(b) $G(s) = \dfrac{1}{(s-5)(s+3)}$

(c) $G(s) = \dfrac{s+1}{s^2}$. For this part, clearly indicate the point at which the closed-loop system has a double-pole.

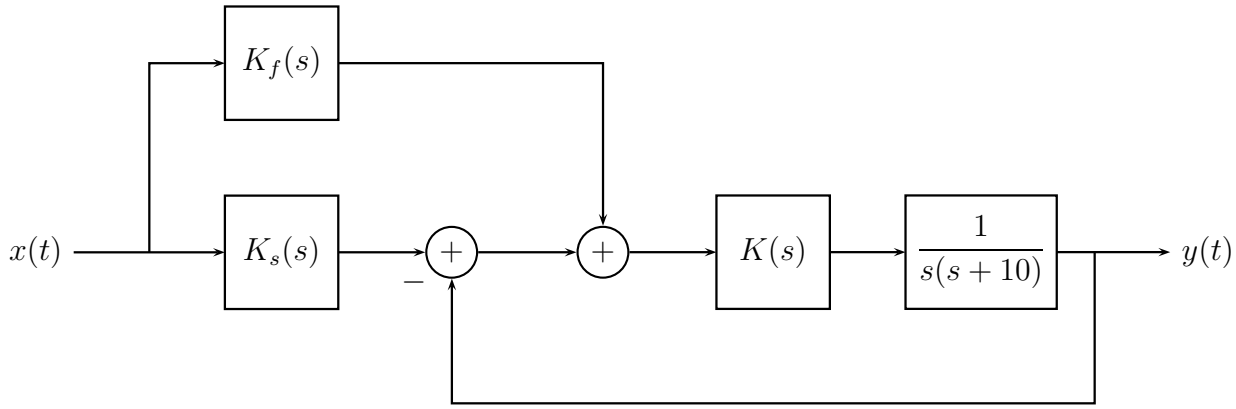
Problem 2 Consider the system shown below:



(a) Compute the steady state tracking error, $e(\infty)$, due to a unit step input $x(t) = u(t)$ when $K(s) = K$. Does the steady state tracking error change as K changes ?

(b) Compute the steady state tracking error, $e(\infty)$, due to a ramp input $x(t) = tu(t)$ when $K(s) = K$. Does the steady state tracking error change as K changes ?

(c) Assume, for this part, that $K(s) = 1$. Find systems $K_f(s)$ and $K_s(s)$ in the modified system shown below such that the steady state tracking error due to the ramp input $x(t) = tu(t)$ becomes zero. *Hint: One of the two systems, $K_f(s)$ and $K_s(s)$, is a constant gain.* Note that the tracking error is defined to be $e(t) = x(t) - y(t)$.



Problem 3 O&W 11.27

Problem 4 Determine the z-transform for each of the following sequences. Sketch pole-zero plot and indicate the region of convergence. Indicate whether or not the Fourier transform of the sequence exists.

(a) $x[n] = 2\delta[n + 3] - \delta[n - 2]$

(b) $x[n] = 2^n u[n - 1] + 4^n u[-n]$

Problem 5 For each of the following z-transforms, determine the inverse z-transform

(a) $X(z) = 12z^{-4} - z^{-1} + 6 + 9z^2 - 8z^5$

(b) $X(z) = \dfrac{5}{1 + \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}, \quad \dfrac{1}{3} < |z| < \dfrac{1}{2}$

Problem 6 Consider a signal $y[n]$ which is related to two signals $x_1[n]$ and $x_2[n]$ by

$$y[n] = x_1[-n-2] * x_2[n+4]$$

where

$$x_1[n] = \left(-\frac{1}{2}\right)^n u[n] \text{ and } x_2[n] = \left(\frac{1}{4}\right)^n u[n].$$

Determine the z-transform $Y(z)$ of $y[n]$, together with its ROC.

Reminder: The first 20 problems in each chapter of O&W have answers included at the end of the text. Consider using these for additional practice, either now or as you study for tests.