

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Electrical Engineering and Computer Science

**6.003: Signals and Systems—Fall 2003**

PROBLEM SET 5

Issued: October 2, 2003

Due: October 22, 2003

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**REMINDER: Quiz 1 will be held from 7:30 to 9:30 p.m. Tuesday, October 14**  
The quiz will cover material in Chapters 1-3 of O&W,  
Lectures and Recitations through September 26, Problem Sets 1-3,  
and that part of Problem Set 4 involving problems from Chapter 3.

**Reading Assignments:**

**Lectures #9-11 & PS#5:** Chapters 4&5 of O&W, plus begin Chapter 6 (through Section 6.1)

**Lectures #12-13 & PS#6:** Chapters 6&7 (through Section 7.2) and Chapter 8 (through Section 8.4) of O&W

**PLEASE NOTE:** The time interval over which you have to work on this problem set is just under three weeks. This is due not only to the fact that there is a quiz in this period, but also, simply to the way the schedule worked out. As a result, this problem set is **longer** than previous problem sets (and subsequent ones) in order to match the coverage of the course. You should use your own judgment in planning the time you devote to this problem set, but it is strongly advised that you not leave it **all** to the last minute.

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**Exercise for home study (not to be turned in, although we will provide solutions):**

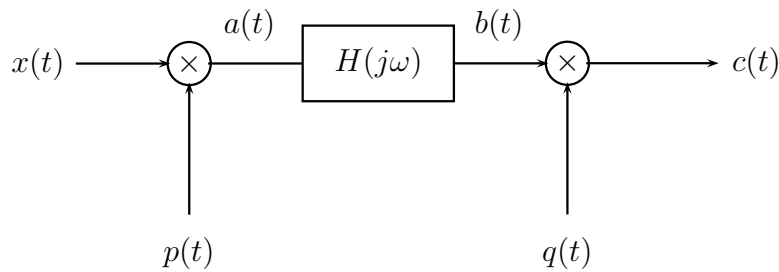
(E1) O&W 4.47

**Problems to be turned in:**

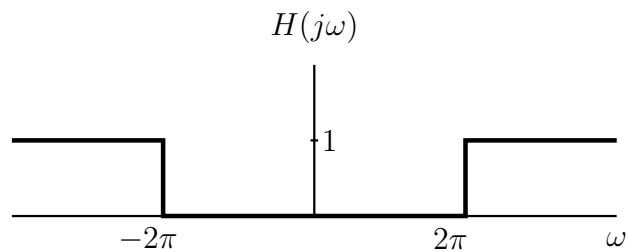
**Problem 1** Consider the signal  $x(t)$  with spectrum depicted in Figure p4.28 (a) of O&W. Sketch the spectrum of

$$y(t) = x(t) [\cos(t/2) + \cos(3t/2)].$$

**Problem 2** Consider the system depicted below:



where  $x(t) = \frac{\sin 4\pi t}{\pi t}$ ,  $p(t) = \cos 2\pi t$ ,  $q(t) = \frac{\sin 2\pi t}{\pi t}$ , and the frequency response of  $H(j\omega)$  is given by



- Let  $A(j\omega)$  be the Fourier transform of  $a(t)$ . Sketch and clearly label  $A(j\omega)$ .
- Let  $B(j\omega)$  be the Fourier transform of  $b(t)$ . Sketch and clearly label  $B(j\omega)$ .
- Let  $C(j\omega)$  be the Fourier transform of  $c(t)$ . Sketch and clearly label  $C(j\omega)$ .
- Compute the output  $c(t)$ .

**Problem 3** O&W 4.44. In addition to parts (a) and (b), answer the following .

- Find the differential equation relating the input and output of this system.

**Problem 4** O&W 5.21 (c), (g)

**Problem 5** The following are Fourier transforms of discrete-time signals. Determine the signal corresponding to each transform.

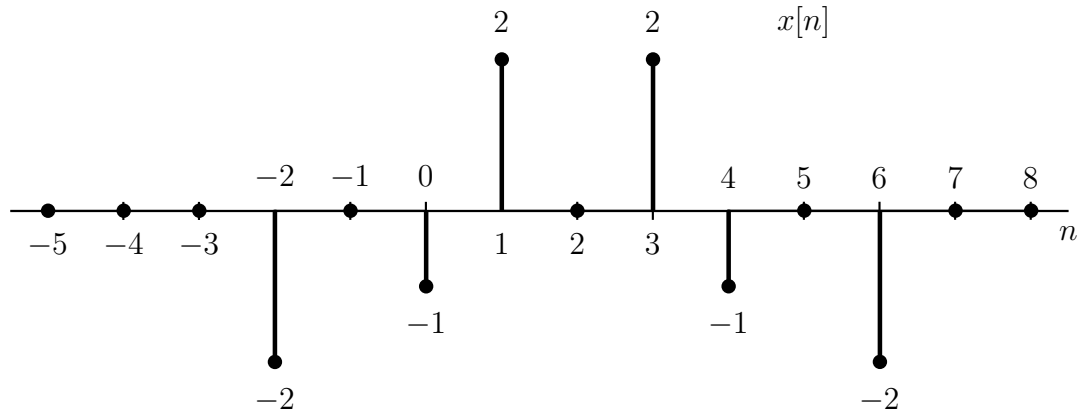
- $X(e^{j\omega}) = 4e^{j4\omega} - e^{j\omega} + 6 + 8e^{-j3\omega} - 16e^{-j11\omega}$

(b)

$$X(e^{j\omega}) = \begin{cases} 1, & 0 \leq |\omega| < \frac{\pi}{4}, \frac{\pi}{2} < |\omega| \leq \pi \\ 0, & \frac{\pi}{4} < |\omega| < \frac{\pi}{2} \end{cases}$$

(c)  $X(e^{j\omega}) = \frac{1 + 3e^{-j3\omega}}{1 + \frac{1}{4}e^{-j\omega}}$

**Problem 6** Let  $X(e^{j\omega})$  denote the Fourier transform of the signal  $x[n]$  depicted below.



(a) Find  $X(1) = X(e^{j0})$ .

(b) Find  $\alpha$  such that  $e^{j\alpha\omega} X(e^{j\omega})$  is real.

(c) Evaluate  $\int_{-\pi}^{\pi} X(e^{j\omega}) d\omega$ .

(d) Find  $X(e^{j\pi})$ .

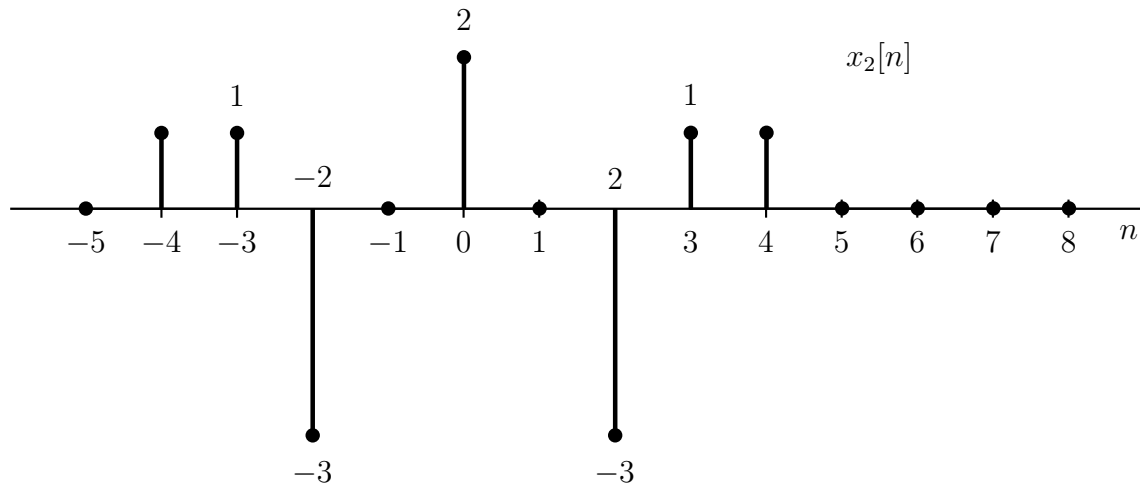
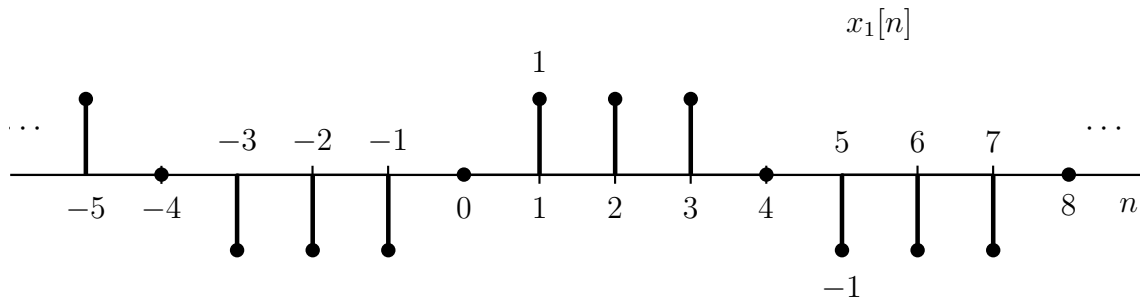
(e) Determine and sketch the signal whose Fourier transform is  $\Re\{X(e^{j\omega})\}$ .

(f) Evaluate each of the following integrals:

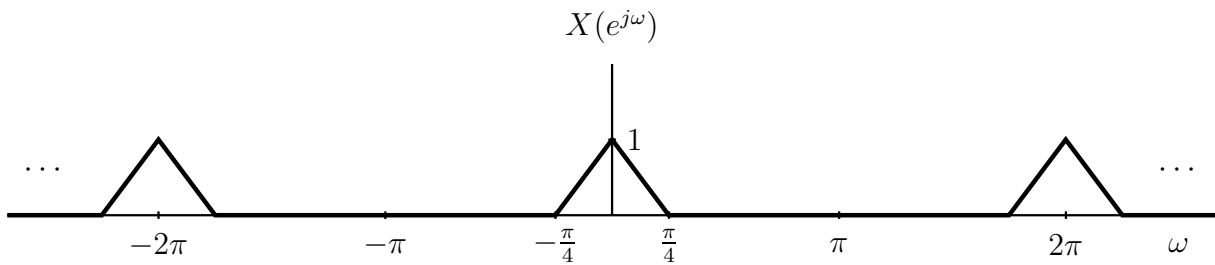
(f.1)  $\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$

(f.2)  $\int_{-\pi}^{\pi} \left| \frac{dX(e^{j\omega})}{d\omega} \right|^2 d\omega$

**Problem 7** Answer the questions asked in O&W 5.24 for the following two signals.



**Problem 8** Consider the same question as asked in O&W 5.27 (a) but with  $X(e^{j\omega})$  as depicted below



and with

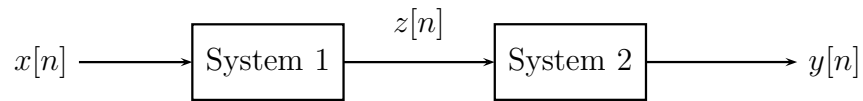
$$p[n] = \cos \pi n - \cos(\pi n/2).$$

**Problem 9** Answer the same questions as asked in O&W 5.30 (b) with  $x[n]$  as given in that problem and for each of the following LTI unit sample responses:

$$(a) \quad h[n] = \frac{\sin \pi n/16}{\pi n} - \frac{\sin(\pi n/12)}{\pi n}$$

$$(b) \quad h[n] = \frac{\sin(\pi n/8) \sin(\pi n/2)}{\pi^2 n^2}$$

**Problem 10** Consider a system consisting of the cascade of two LTI systems as depicted below



System 1 is LTI and has a unit-sample response of

$$h_1[n] = \left(\frac{1}{4}\right)^n u[n].$$

System 2 is also LTI, and we know that if the input is

$$z[n] = \delta[n] + \frac{1}{2}\delta[n - 1]$$

the output is

$$y[n] = 10\delta[n] - \delta[n - 1].$$

In the following parts, please show your work.

- What is the frequency response  $H(e^{j\omega})$  of the overall system ?
- Find the difference equation for the overall system.
- Find the impulse response of the overall system.

**Reminder:** The first 20 problems in each chapter of O&W have answers included at the end of the text. Consider using these for additional practice, either now or as you study for tests.