Lecture 5 - Non-uniformly Doped Samples in TE - Outline

- **Announcements**
  - Handout - Lecture Outline and Summary
- **Review**
  - Non-uniform injection (flow) problems: (Using p-type as the example)
    - Meeting 5 flow problem conditions yields one equation for $n'(x)$
      - $d^2n'/dx^2 - n'/L_e^2 = - g_L(x)/D_e$ with $L_e \equiv (D_e t_e)^{1/2}$
    - Knowing $n'(x)$, then $J_e(x)$, $J_h(x)$, $E(x)$, and $p'(x)$ follow
    - Can/should check that flow problem conditions are met
  - Infinite lifetime limit/approximation
    - L_{min} >> sample dimension (equivalent to t_e becoming infinite)
      - $n'(x) \approx \int\int [- g_L(x)/D_e]\, dx\, dx$
- **Non-uniformly doped samples in thermal equilibrium**
  - One condition, i.e., thermal equilibrium:
    - $J_e(x) = J_h(x) = 0$; leaves 3 equations in $n_o(x)$, $p_o(x)$, and $E(x)$
  - $n_o(x)$ and $p_o(x)$ as functions of TE electrostatic potential, $f_o(x)$
  - The Einstein Relation linking D and μ
  - Poisson's equation for $f_o(x)$ given $N_d(x)$ and $N_a(x)$
- **Solving Poisson's equation in special cases**
  - Slowly varying profiles - quasi-neutrality holds
  - Abrupt p-n junctions - the Depletion Approximation

A general description of semiconductor devices

(Modeling carriers in isothermal semiconductor devices)

The Five Basic Equations of Semiconductor Physics

Electron population

$$\frac{\partial n(x,t)}{\partial t} - \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} = g_L(x,t) - \left[n(x,t)\ p(x,t) - n_i^2\right] r(T)$$

Hole population

$$\frac{\partial p(x,t)}{\partial t} + \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x} = g_L(x,t) - \left[n(x,t)\ p(x,t) - n_i^2\right] r(T)$$

Electron current density

$$J_e(x,t) = q\mu_e n(x,t) E(x,t) + qD_e \frac{\partial n(x,t)}{\partial x}$$

Hole current density

$$J_h(x,t) = q\mu_h p(x,t) E(x,t) - qD_h \frac{\partial p(x,t)}{\partial x}$$

Poisson's equation

$$\frac{\partial E(x,t)}{\partial x} = \frac{q}{\varepsilon}\left[p(x,t) - n(x,t) + N_d^+(x) - N_a^-(x)\right]$$

Flow Problems: a subset of situations described by the five equations

(Minority carrier flow [diffusion] is an important element of all junction devices.)

Flow Problems: When five conditions are met

1. **Uniformly doped, extrinsic material**
2. **Low level injection**
3. **Quasi-neutral**
4. **Negligible minority carrier drift**
5. **Quasi-static excitation**

then we have a problem that qualifies as a "Flow Problem," and the minority carrier population satisfies the following equation:

$$\frac{d^2 n'(x)}{dx^2} - \frac{n'(x)}{D_e t_e} = -\frac{1}{D_e} g_L(x)$$

The solution of a flow problem involves first finding $n'(x)$ **by finding the homogeneous and particular solutions to this second order differential equation, and then matching their sum to the boundary conditions.**

Flow Problems, cont.: **Solving the steady state diffusion equation gives n′. Finding p′, J_e, J_h, and E_x knowing n′:**

J_e: $$J_e(x) \approx qD_e \frac{dn'(t)}{dx}$$

J_h: $$J_h(x) = J_{Tot} - J_e(x)$$

E_x: $$E_x(x) \approx \frac{1}{q\mu_h p_o}\left[J_h(x) - \frac{D_h}{D_e} J_e(x)\right]$$

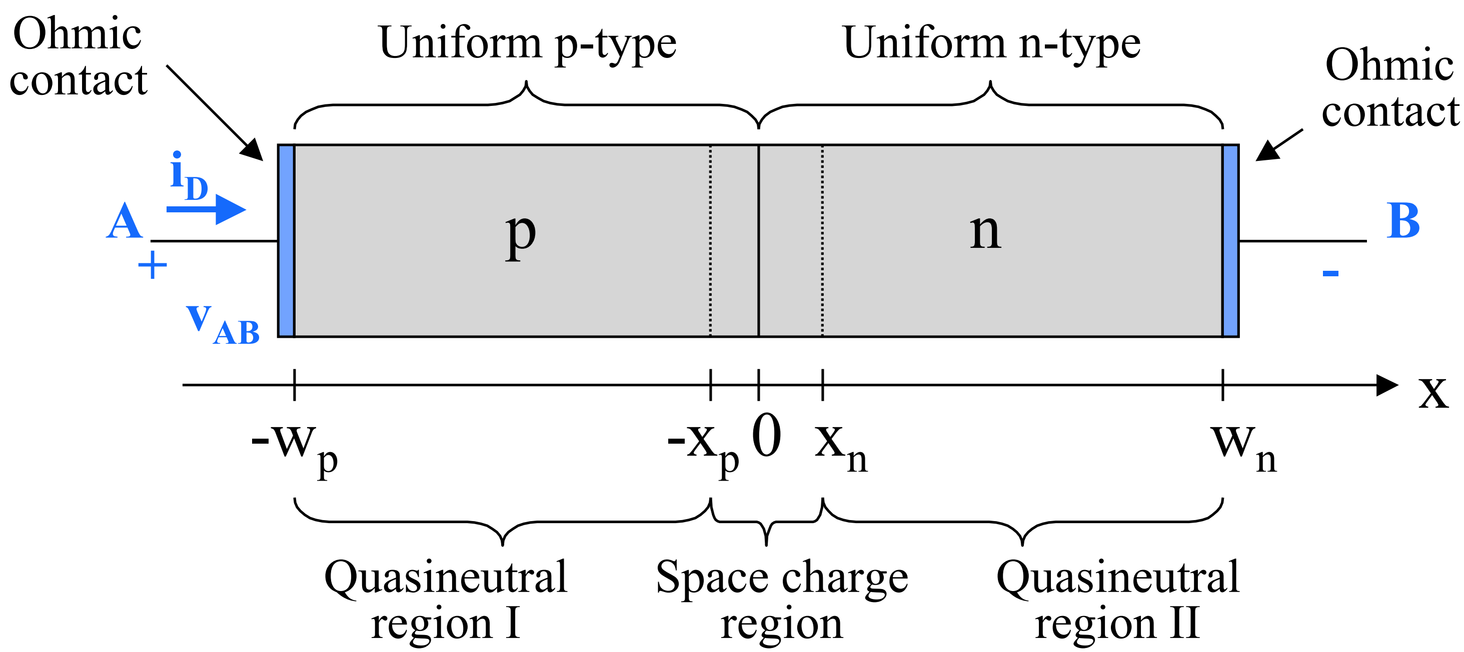
p′: $$p'(x) \approx n'(x) + \frac{\varepsilon}{q}\frac{dE_x(x)}{dx}$$

Note: After determining all of the 5 unknowns, one can (and should, but seldom does) go back and check that all of the five conditions are met by the solution.

The p-n Junction Diode:

Our goal is to determine the relationship between i_D and v_{AB}.

A p-n junction diode has three regions (see below) and we already know how to deal with the quasineutral regions.



Flow problem, QN region I - Excitation: $g_L = 0$

B.C.'s: $n'(-w_p) = 0$, $n'(-x_p)$ = function* of v_{AB}

Flow problem, QN region II - Excitation: $g_L = 0$

B.C.'s: $p'(w_n) = 0$, $p'(-x_n)$ = function* of v_{AB}

*** Getting this function is our next topic.**

Doping Profile Problems: another special case

(Doping profiles and junctions are essential components of any semiconductor device)

Doping Profiles and p-n Junctions in TE:

To model devices we need to introduce changes in the level and type of doping.

To treat non-uniformly doped materials we begin by looking at them in thermal equilibrium.

We can immediately say:

$g_L(x,t) = 0$

$n(x) = n_o(x)$

$p(x) = p_o(x)$

$J_e(x) = 0$

$J_h(x) = 0$

$\partial/dt = 0$

Furthermore, the first two of our five equations reduce to 0 = 0, e.g.:

$$\frac{\partial n(x,t)}{\partial t} - \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} = g_L(x,t) - \left[n(x,t)\;\; p(x,t) - n_o(x)\;\; p_o(x)\right] r(T)$$

0 0 0 0

Doping Profiles and p-n Junctions in TE, cont.:

The third and fourth equations, the current equations give:

$$0 = q\mu_e n_o(x)E(x) + qD_e \frac{dn_o(x)}{dx} \quad \Rightarrow \quad \frac{d\phi}{dx} = \frac{D_e}{\mu_e}\frac{1}{n_o(x)}\frac{dn_o(x)}{dx}$$

$$0 = q\mu_h p_o(x)E(x) - qD_h \frac{dp_o(x)}{dx} \quad \Rightarrow \quad \frac{d\phi}{dx} = -\frac{D_h}{\mu_h}\frac{1}{p_o(x)}\frac{dp_o(x)}{dx}$$

And Poisson's equation becomes:

$$\frac{dE(x)}{dx} = -\frac{d^2\phi(x)}{dx^2} = \frac{q}{\varepsilon}\left[p_o(x) - n_o(x) + N_d(x) - N_a(x)\right]$$

In the end, we have three equations in our three remaining unknowns, $n_o(x)$, $p_o(x)$, and $\phi(x)$, so all is right with the world.

Lecture 5 - Non-uniformly Doped Samples in TE - Summary

- **Non-uniformly doped samples in thermal equilibrium**
  - One condition must be met, i.e., thermal equilibrium:
    - $J_e(x) = J_h(x) = 0$; drift and diffusion are balanced
    - Leaves 3 equations in $n_o(x)$, $p_o(x)$, and $E_o(x)$
  - $n_o(x)$ and $p_o(x)$ as functions of electrostatic potential, $\phi_o(x)$:
    - $n_o(x) = n_i \exp[q\phi_o(x)/kT]$, $p_o(x) = n_i \exp[-q\phi_o(x)/kT]$
      - [Note: continue to have $n_o(x)\, p_o(x) = n_i^2$]
  - The Einstein Relation: $D/\mu = kT/q$ or $\mu/D = q/kT$ (Either way, it rhymes)
  - Poisson's equation for $\phi_o(x)$ given $N_d(x)$ and $N_a(x)$:
    - $\epsilon\, d^2\phi_o(x)/dx^2 = q\,[n_i\,\{e^{q\phi_o(x)/kT} - e^{-q\phi_o(x)/kT}\} - N_d(x) + N_a(x)]$
    - Very non-linear; cannot in general be solved analytically
- **Solving Poisson's equation in two special cases**
  - Slowly varying profiles - quasi-neutrality holds
    - [Note: "Slowly" means in a Debye length, $L_D \{= (\epsilon kT/q^2 N_A)^{1/2}\}$, $\Delta\phi_o < kT$]
    - In n-type samples, $n_o(x) \approx N_d(x) - N_a(x)$, $p_o(x) = n_i^2/n_o(x)$
    - In p-type samples, $p_o(x) \approx N_a(x) - N_d(x)$, $n_o(x) = n_i^2/p_o(x)$
    - In both cases, $\phi_o(x) = (kT/q)\ln[n_o(x)/n_i] = -(kT/q)\ln[p_o(x)/n_i]$
    - 60mV rule: ϕ_o changes 60mV for every factor of 10 change in n_o, p_o
  - Abrupt p-n junctions - the Depletion Approximation
    - Very large doping gradients and net charge densities
    - Drift and diffusion balance (The topic of Lecture 6)