6.012 - Electronic Devices and Circuits
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MOSFET Drain Current Modeling

In the Gradual Channel Model for the MOSFET we write the drain current, i_D, as the product of $q_N^*(y)$, the inversion layer sheet charge density at position y along the channel; $s_y(y)$, the net drift velocity of the inversion layer carriers there (electrons in the n-channel device we are modeling), and W, the channel width:

$$i_D = -q_N^*(y)\ s_y(y)\, W$$

with

$$q_N^*(y) = -\frac{\varepsilon_{ox}}{t_{ox}}[v_{GB} - V_T(y)] \quad \text{and} \quad s_y(y) = -\mu_e E_y = \mu_e \frac{dv_{CS}(y)}{dy}$$

Substituting these expressions yields:

$$i_D = W\mu\, C_{ox}^*\, [v_{GB} - V_T(y)]\frac{dv_{CS}(y)}{dy}$$

where we have identified the gate capacitance per unit area, C_{ox}^*, as ε_{ox}/t_{ox} and where the threshold voltage is given by

$$V_T(y) = V_{FB} + |2\phi_p| + v_{CB}(y) + \frac{1}{C_{ox}^*}\sqrt{2\varepsilon_{Si}\, qN_A\, [\,|2\phi_p| + v_{CB}(y)}$$

Defining the body factor, γ, as $\sqrt{2\varepsilon_{Si}\, qN_A}\ /C_{ox}^*$, and writing $v_{CB}(y)$ as $v_{CS}(y) - v_{BS}$, we can rewrite this as

$$V_T(y) = V_{FB} + |2\phi_p| + v_{CS}(y) - v_{BS} + \gamma\sqrt{|2\phi_p| + v_{CS}(y) - v_{BS}}$$

and thus we can write i_D as

$$i_D = W\mu_e C_{ox}^*\ [v_{GS} - V_{FB} - |2\phi_p| - v_{CS}(y) - \gamma\sqrt{|2\phi_p| + v_{CS}(y) - v_{BS}}\ ]\frac{dv_{CS}(y)}{dy}$$

To proceed we integrate both sides for y = 0 to y = L, recognizing that the right-hand integral is equivalent to integrating with respect to $v_{CS}(y)$ from 0 to v_{DS}:

$$i_D \int_0^L dy \; = W \mu_e C_{ox}^* \int_0^{v_{DS}} [v_{GS} - V_{FB} - |2\phi_p| - v_{CS} - \gamma\sqrt{|2\phi_p| + v_{CS} - v_{BS}}\,] \; dv_{CS}$$

The left-hand integral is $i_D L$, so we can write i_D as

$$i_D = K \int_0^{v_{DS}} [v_{GS} - V_{FB} - |2\phi_p| - v_{CS} - \gamma\sqrt{|2\phi_p| + v_{CS} - v_{BS}}\,] \; dv_{CS}$$

where K is defined as (W/L) $\mu_e C_{ox}^*$.

It is not hard to do the integral on the right-hand side of this equation, and you may want to do it as an exercise (it is done in the text and the result is given in Equation 10.9). The resulting expression is awkward and, most importantly, the threshold voltage, V_T, is hard to identify in the expression and the role it plays in the current-voltage relationship is hard to understand; the result is not very intuitive. It will not be obvious to you until you get much more experience with MOSFETs, but it is very desirable from a modeling standpoint to do something to simplify the result and to get an expression that gives us more useful insight.

Many texts simply ignor the v_{CS} factor under the radical and write

$$i_D \quad K \int_0^{v_{DS}} [v_{GS} - V_{FB} - |2\phi_p| - v_{CS} - \gamma\sqrt{|2\phi_p| - v_{BS}}\,] \; dv_{CS}$$

which we can simplify as

$$i_D = K \int_0^{v_{DS}} [v_{GS} - V_T' - v_{CS}\,] \; dv_{CS}$$

with V_T' defined to be $V_{FB} + |2\phi_p| + \gamma\sqrt{|2\phi_p| - v_{BS}}$. Doing the integral we get

$$i_D = K\,[(v_{GS} - V_T')v_{DS} - \frac{v_{DS}^2}{2}\,]$$

A more satisfying approach is to not ignor the v_{CS} factor, but rather to try to linearize the dependance on it. The troublesome term is

$$\sqrt{|2\phi_p| + v_{CS}(y) - v_{BS}}$$

which can be written as

$$\sqrt{|2\phi_p| + v_{CS} - v_{BS}} \quad = \sqrt{|2\phi_p| - v_{BS}}\sqrt{1 + \frac{v_{CS}}{|2\phi_p| - v_{BS}}}$$

$$\sqrt{|2\phi_p| - v_{BS}} \quad \left[1 + \frac{v_{CS}}{2(|2\phi_p| - v_{BS})}\right]$$

$$\sqrt{|2\phi_p| - v_{BS}} \quad + \frac{v_{CS}}{2\sqrt{|2\phi_p| - v_{BS}}}$$

With this approximation, we next define $1/2\sqrt{|2\phi_p| - v_{BS}}$ to be δ and $(1 + \gamma\delta)$ to be α, and write i_D as

$$i_D \quad K\int_0^{v_{DS}} [v_{GS} - V_{FB} - |2\phi_p| - \alpha\, v_{CS} - \gamma\sqrt{|2\phi_p| - v_{BS}}\,] \; dv_{CS}$$

Using our earlier definition for V_T', this becomes

$$i_D \quad K\int_0^{v_{DS}} [v_{GS} - V_T' - \alpha\, v_{CS}\,] \; dv_{CS}$$

and doing the integral yields

$$i_D = K\left[(v_{GS} - V_T')v_{DS} - \alpha\frac{v_{DS}^2}{2}\right]$$

In saturation, which now occurs for $v_{DS} > (v_{GS} - V_T')/\alpha$, we have

$$i_D = \frac{K}{2\alpha}(v_{GS} - V_T')^2$$

These results are the same as those we obtained after ignoring v_{CS} under the radical, except that we now have a factor of α appearing. To the extent that α is very near one, our earlier approximation is correct, inspite of it being rather adhoc. Collecting all the factors in α, we find it is

$$\alpha = 1 + \frac{\sqrt{2\varepsilon_{Si}\, q N_A}}{2\, C_{ox}^{*}\sqrt{|2\phi_p| - v_{BS}}}$$

Typically this is near 1, and it can be approximated as such. On the other hand, it is easy to leave α in the expression for i_D since is such a minor complication.

To summarize, our expressions for the drain current, when we retain α are

$$i_D = 0 \qquad \text{for } (v_{GS} - V_T')/\alpha < 0 < v_{DS} \qquad \text{(Cutoff)}$$

$$i_D = \frac{K}{2\alpha}(v_{GS} - V_T')^2 \qquad \text{for } 0 < (v_{GS} - V_T')/\alpha < v_{DS} \qquad \text{(Saturation)}$$

$$i_D = K\left[(v_{GS} - V_T')v_{DS} - \alpha\frac{v_{DS}^2}{2}\right] \text{ for } 0 < v_{DS} < (v_{GS} - V_T')/\alpha \qquad \text{(Linear region)}$$

with K, V_T', γ, and α defined as

$$K \equiv (W/L)\,\mu_e\, C_{ox}^*$$

$$V_T' \equiv V_{FB} + |2\phi_p| + \gamma\sqrt{|2\phi_p| - v_{BS}}$$

$$\gamma \equiv \frac{\sqrt{2\varepsilon_{Si}\, qN_A}}{C_{ox}^*}$$

$$\alpha \equiv 1 + \frac{\sqrt{2\varepsilon_{Si}\, qN_A}}{2\, C_{ox}^*\sqrt{|2\phi_p| - v_{BS}}}$$