

## Exam No. 1, Fall 2001, Answers

**Problem 1:**

(a) Boron is an acceptor and phosphorous is a donor, so there is a net donor concentration of  $10^{16} \text{ cm}^{-3}$ . Thus the sample is n-type, and the equilibrium electron concentration is  $10^{16} \text{ cm}^{-3}$ . The equilibrium hole concentration is  $n_i^2/n_o = 10^4 \text{ cm}^{-3}$ . The electrostatic potential is approximately  $6 \times 0.060 \text{ V} = 0.36 \text{ V}$ .

(b) The minority carrier diffusion lengths are  $(D\tau)^{1/2}$  so we first have to find  $D = kT\mu/q$ . We find  $D_e = 40 \text{ cm}^2/\text{s}$  and  $D_h = 15 \text{ cm}^2/\text{s}$ , so  $L_e = 2 \times 10^2 \text{ cm}$  and  $L_h = 1.2 \times 10^2 \text{ cm}$ . (Opps: Electrons are not minority carriers in this sample and asking for  $L_e$  may have been confusing to some of you; in retrospect we should have anticipated this.)

(c) The resistivity is dominated by the majority carrier so  $\rho_o = 1/(qn_o\mu_e)$ , from which we find  $n_o = 1/(q\rho_o\mu_e) = 4 \times 10^{15} \text{ cm}^{-3}$ , and  $p_o = n_i^2/n_o = 2.5 \times 10^4 \text{ cm}^{-3}$ .

(d) With steady-state illumination, the excess populations,  $n'$  and  $p'$ , are  $G_L\tau_{\min}$ , if LLI holds. Assuming it does we find  $n' = p' = 10^{19} \text{ cm}^{-3}\text{s}^{-1} \times 10^{-5} \text{ s} = 10^{15} \text{ cm}^{-3}$ , from which we also see that our assumption of LLI is valid. Since the sample is p-type and LLI holds,  $p$  is essentially  $p_o (= N_A)$ , or  $10^{17} \text{ cm}^{-3}$ , and  $n$  is approximately  $n'$ , or  $10^{15} \text{ cm}^{-3}$ .

The change in conductivity is  $q(\mu_e + \mu_h)n'$ , which works out to be  $0.35 \text{ S/cm}$ .

(e) The insights (also called "tricks") are to note (1) that the average population is  $\tau_{\min}$  times the average generation, and (2) that if the time variation is slow relative to minority carrier lifetime, the time varying portion of the excess population is  $\tau_{\min}$  times the time varying generation, all assuming LLI holds, of course. (Also note that the minority carrier population is essentially the excess minority carrier population because the equilibrium population is so small.)

Since the average generation is the same as in Part d, we know  $n_{\text{ave}}$  is  $10^{15} \text{ cm}^{-3}$ . Already we can also see that because the peak generation rate is only twice the average, LLI will hold.

Turning then to the time varying excitation, we see that the period is  $10^{-2} \text{ s}$ , with is much greater than the minority carrier lifetime of  $10^{-5} \text{ s}$ , so we have that  $n(t) - n_{\text{ave}} = [g_L(t) - g_{L,\text{ave}}]\tau_{\min} = 10^{15}\sin(2 \times 10^2 t) \text{ cm}^{-3}$ .

**Problem 2:**

(a) Just inside the bar the electron flux must be the same as it is in the incident electron beam, or  $10^{19}/\text{cm}^2\text{-s}$ . Thus the current density is this flux multiplied by  $-1.6 \times 10^{-19} \text{ coul/electron}$ ; doing this we find  $J_e(0^+) = -1.6 \text{ A/cm}^2$ .

(b)  $n'(x) = n'(0)e^{-x/L_e}$ . We find  $n'(0)$  by insisting that this expression give us the correct value for  $J_e(0^+)$ . We can get an expression for  $J_e(0^+)$  by recalling that it is  $qD_e dn'/dx|_{x=0}$ , and using our formula for  $n'(x)$ . Doing this we find that we must have  $qD_e n'(0)/L_e = 1.6$  and thus  $n'(0) = 1.6 L_e/qD_e = (1.6 \times 10^{-3})/(1.6 \times 10^{-19})(40) = 2.5 \times 10^{14} \text{ cm}^{-3}$ .

(c) The electron current density decreases with  $x$  in the same exponential manner as  $n'(x)$  and thus, since we already know  $J_e(0^+)$ , we can write  $J_e(x) = -1.6 e^{-x/L_e} \text{ A/cm}^2$ .

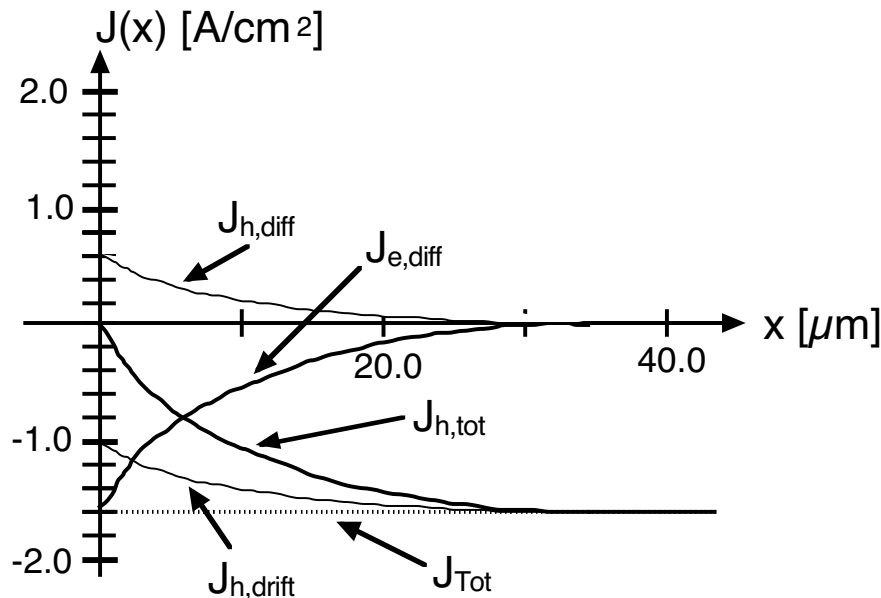
(d) The hole current density is  $J_{TOT} - J_e(x)$  and we know  $J_{TOT}$  because we know the total current density one place, namely in the region of the electron beam. In this region the total current density is  $-1.6 \text{ A/cm}^2$ , as we calculated in Part a. Thus  $J_h(x) = J_{TOT} - J_e(x) = -1.6 (1 - e^{-x/L_e}) \text{ A/cm}^2$ .

(e) The electric field is found from the majority current,  $J_{h,drift}(x) = J_{h,total}(x) - J_{h,diff}(x) = q\mu_h p_o E(x)$ , and using quasineutrality,  $dp'/dx = -dn'/dx$ , which together allow us to write  $E(x) = \{J_{h,total}(x) + (D_h/D_e)J_e(x)\}/q\mu_h p_o$ . It is convenient to first calculate  $1/q\mu_h p_o$ , which we find is approximately  $0.1 \text{ Ohm-cm}$ . Putting this all together we find:

$$E(x) = -1.6 [1 - (5/8) e^{-x/L_e}] (0.1) = -0.16 + 0.1 e^{-0.001x} \text{ V/cm}.$$

(f) We find the voltage drop by integrating the electric field from  $x = 0$  to  $x = 100 \mu\text{m}$ . Doing this we find  $\Delta\phi = -(0.16)(0.01) + (0.1)(0.001) = -1.5 \text{ mV}$ .

It is interesting to compare this result with the voltage drop in a bar of p-type Si like this with ohmic contacts at each end and in which the current density is a constant  $-1.6 \text{ A/cm}^2$  (all hole drift current):  $\Delta v = \rho J L = (0.1 \text{ Ohm-cm})(-1.6 \text{ A/cm}^2)(10^{-2} \text{ cm}) = -1.6 \text{ mV}$ . The potential drop is slightly smaller in the first case because the magnitude of the hole drift current is less than  $1.6 \text{ mA}$  on the left end of the bar. This can most easily be seen by plotting the various current components, which is done below (you weren't asked to do this):



### Problem 3:

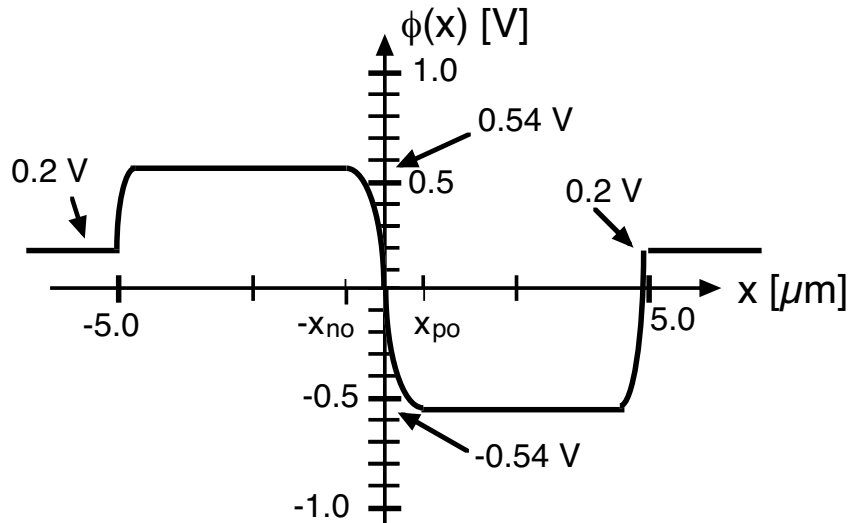
(a) On the n-side the electrons are the majority carrier and thus  $n_{no} = N_D = 10^{15} \text{ cm}^{-3}$ . The equilibrium minority carrier concentration is  $n_i^2/n_o$  so  $p_{no} = n_i^2/N_D = (10^6)^2/10^{15} \text{ cm}^{-3} = 10^{-3} \text{ cm}^{-3}$ . Remember, this is the density of holes, not the number of holes, and a density can be less than one.

(b) The expression for built-in potential is the same as always, and thus  $\phi_o = (kT/q) \ln(N_{Dn}N_{Ap}/n_i^2) = 0.06 \text{ V} \log(10^{15}10^{15}/10^{12}) = 0.06 \text{ V} \log(10^{18}) = 1.08 \text{ V}$ .

This is much greater than in a comparably doped silicon diode because of the much smaller intrinsic concentration. Remember, a little potential goes a long way in the exponent.

(c) The zero-bias depletion width will be greater in this diode than in a comparably doped silicon diode because the potential step is almost twice as large. The ratio of the widths will be the square root of the ratio of built-in potentials, or  $(1.08/0.6)^{1/2} = 1.34$ .

(d) Beyond remembering that the n-side is on the left now, rather than on the right as we usually draw p-n diodes, and using the relevant values for the electrostatic potential in the n- and p-regions, the problem is rather pedestrian (sorry about that):



(e) (i) In any junction the excess minority carrier population at the edge of the depletion region in the quasineutral region is the minority population times  $(e^{qV_{BA}/kT} - 1)$ . With  $v_{AB} = 0.6$  V, this factor is approximately  $10^{10}$  if we use the "60 mV rule" approximation, or  $2.7 \times 10^{10}$  if we use  $kT/q = 25$  mV (either is acceptable); we found  $p_{no} = 10^{-3} \text{ cm}^{-3}$  in Part a, so combining these we can find  $p'(-x_n) = 2.7 \times 10^7 \text{ cm}^{-3}$ .

Based on the devices symmetry, we must have  $n'(x_p) = p'(-x_n) = 2.7 \times 10^7 \text{ cm}^{-3}$ .  
(These answers are  $n'(x_p) = p'(-x_n) = 10^7 \text{ cm}^{-3}$  if we use  $10^{10}$ , i.e. the 60 mV rule.)

(ii) One can plug into the diode equation, with  $I_s$  spelled out in terms of the diode dimensions, etc., but it is quicker knowing the excess as we do in Part e to write:

$$J_e(0) = -q D_e n'(x_p) / w_p = -(1.6 \times 10^{-19})(8 \times 10^1)(2.7 \times 10^7) / (5 \times 10^{-4}) = -6.8 \times 10^{-7} \text{ A/cm}^2$$

$$J_h(0) = -q D_h p'(-x_n) / w_n = -(1.6 \times 10^{-19})(15)(2.7 \times 10^7) / (5 \times 10^{-4}) = -1.3 \times 10^{-7} \text{ A/cm}^2$$

$$\text{Finally, we have } J_{tot} = J_e(0) + J_h(0) = -8.1 \times 10^{-7} \text{ A/cm}^2$$

[Using the 60 mV rule the values are:  $J_e(0) = -2.56 \times 10^{-7} \text{ A/cm}^2$ ,  $J_h(0) = -4.8 \times 10^{-8} \text{ A/cm}^2$ , and  $J_{tot} = -3 \times 10^{-7} \text{ A/cm}^2$ ]

(f) (i) If the p-side was silicon, then the excess electron population at  $x_p$ , i.e.,  $n'(x_p)$ , would be the equilibrium electron concentration in p-type silicon doped with  $10^{15} \text{ cm}^{-3}$  acceptors, or  $10^5 \text{ cm}^{-3}$ , multiplied by the same  $(e^{qV_{BA}/kT} - 1)$  factor of  $2.7 \times 10^{10}$ , meaning now  $n'(x_p) = 2.7 \times 10^{15} \text{ cm}^{-3}$  (or simply  $10^{15} \text{ cm}^{-3}$  if you used the 60 mV rule).

Unfortunately the numbers were not chosen so this does not violate our low level injection assumption, but even in spite of this complication we clearly see that now  $J_e(0)$  will be much higher than it was in our original device. If we apply our low level injection equation for the diode current (which the person who made up the problem had intended you would be able to do), we find

$$J_e(0) = -qD_e n'(x_p)/w_p = -(1.6 \times 10^{-19})(4 \times 10^{17})(2.7 \times 10^{15})/(5 \times 10^{-4}) = -3.4 \times 10^1 \text{ A/cm}^2$$

Comparing this to  $J_h(0)$ , which is unchanged from before, we find that the injection across the junction is predominantly electrons, even though the junction is symmetrically doped:  $J_e(0)/J_h(0) = 2.6 \times 10^8$ .

(ii) The use of a junction between two different semiconductors is one way of being able to engineer the injection across a junction, as one wants to do at the emitter-base junction of a bipolar transistor, for example, without having to have one side much more heavily doped than the other.

**Who graded what:**

Problem 1	Prof. Ram
Problem 2	Prof. Fonstad
Problem 3	Prof. Gray

<b><u>Statistics:</u></b>	<b>Ave.</b>	<b>St. Dev.</b>
Problem 1	28.3	4.1
Problem 2	22.7	7.7
<u>Problem 3</u>	<u>24.7</u>	<u>7.3</u>
Total Exam	75.7	16.5

**The grades distribution:** Can you find yourself in this picture?

