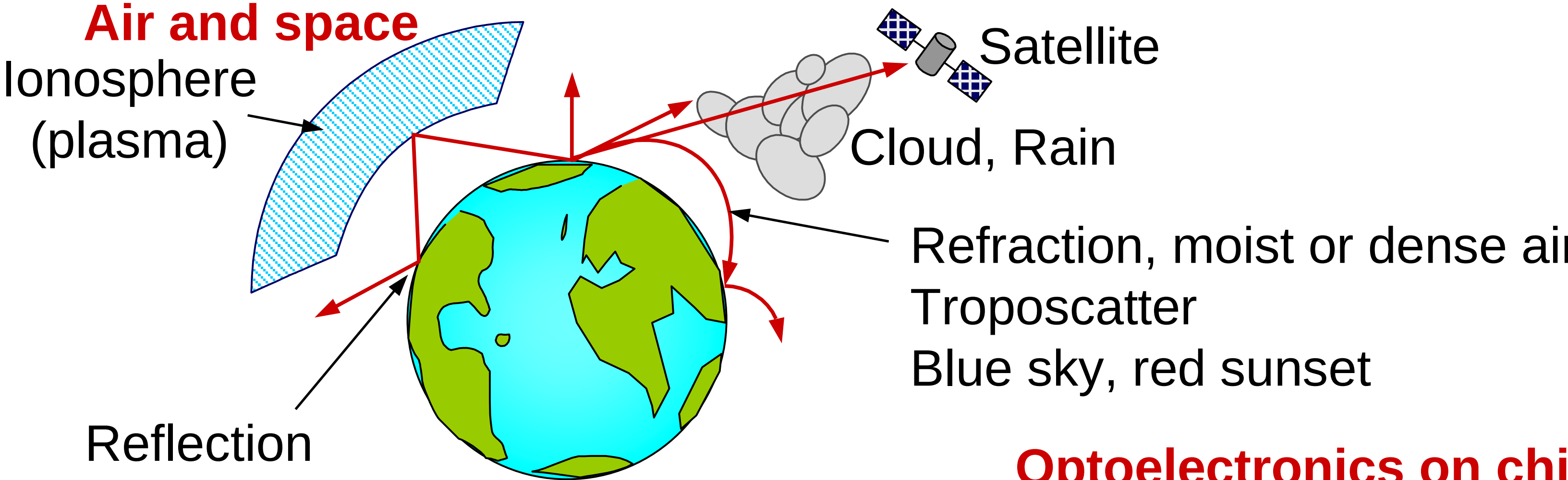
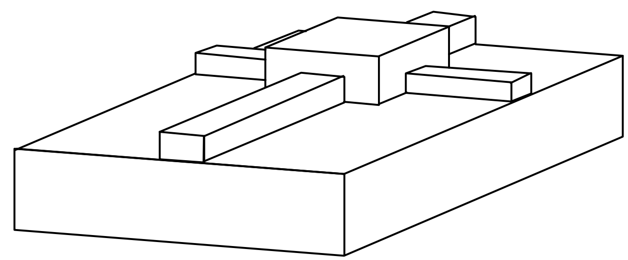
WAVES IN MEDIA

Significance to Communications

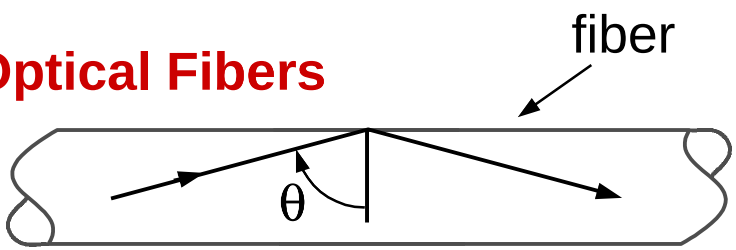
Air and space



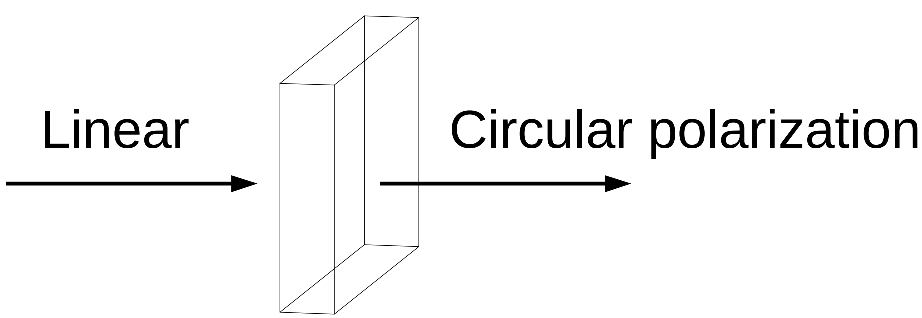
Optoelectronics on chips



Optical Fibers

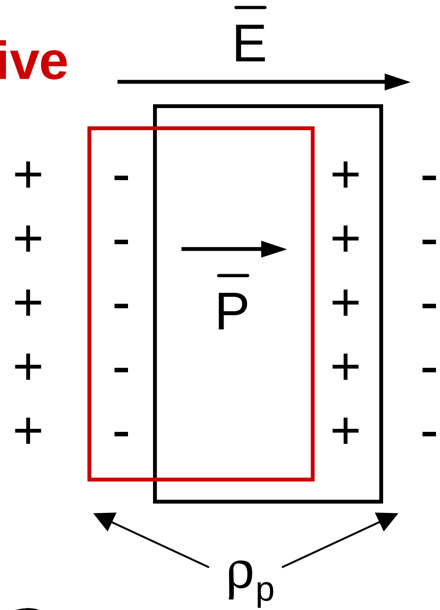


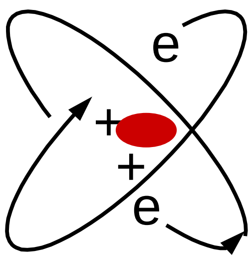
Polarization-based optoelectronic devices

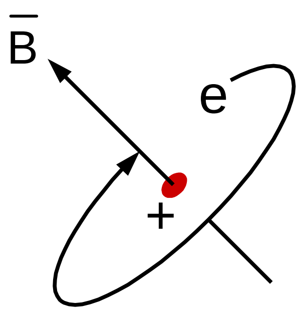


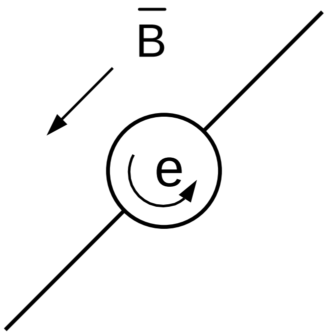
WAVES IN MEDIA

Constitutive Relations









Vacuum:

$$\overline{D} = \varepsilon_o \overline{E} \qquad \nabla \bullet \overline{D} = \rho_f$$

ρ_f = free charge density

Dielectric Materials:

$$\overline{D} = \varepsilon \overline{E} = \varepsilon_o \overline{E} + \overline{P}$$

$$\nabla \bullet \varepsilon_o \overline{E} = \rho_f + \rho_p$$

$\nabla \bullet \overline{P} = -\rho_p$ polarization charge density

$\overline{P}$ = "Polarization Vector"

Magnetic Materials:

$$\nabla \bullet \overline{B} = 0$$

$\overline{B} = \mu_o \overline{H}$ in vacuum

$$\overline{B} = \mu \overline{H} = \mu_o \left(\overline{H} + \overline{M} \right)$$

$\overline{M}$ = "Magnetization Vector"

TYPES OF MEDIA

Properties are a function of:

Field direction

Position

Time: $\neq f(t)$

$\neq f(\text{history})$

Frequency

$\overline{E}$ or $\overline{H}$

Temperature

Pressure

Designation:

Anisotropic $\underline{\overline{D}} = \underline{\overline{\overline{\varepsilon}}}\,\underline{\overline{E}}$, $\underline{\overline{B}} = \underline{\overline{\overline{\mu}}}\,\underline{\overline{H}}$

Inhomogeneous

Stationary

Amnesic

Dispersive

Non-linear

Temperature dependent

Compressive

ANISOTROPIC DIELECTRICS

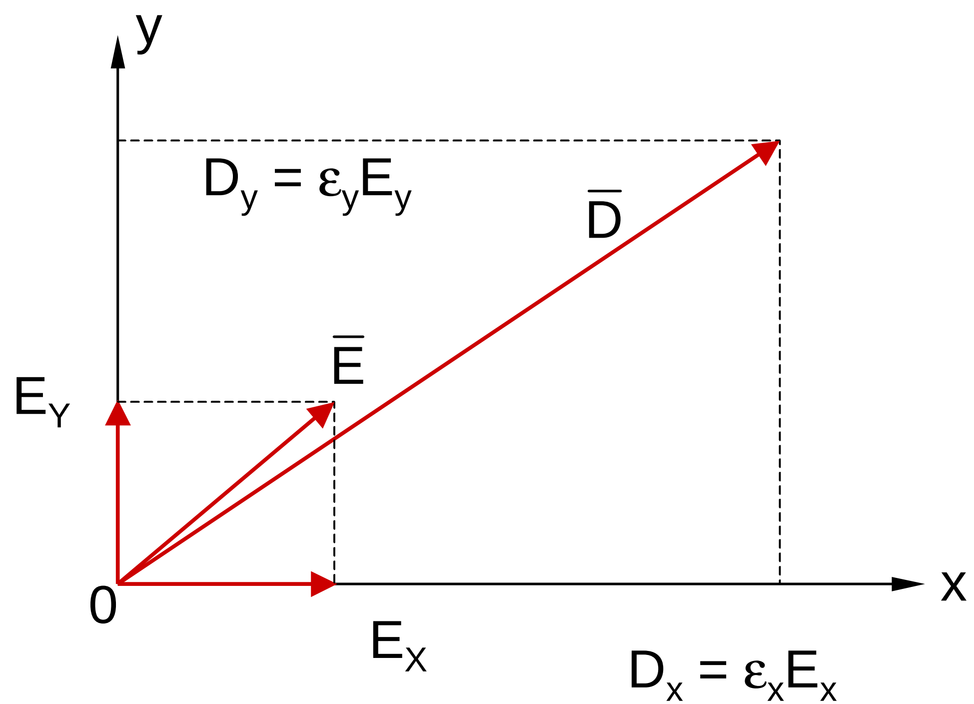
$$\underline{\overline{D}} = \underline{\underline{\overline{\overline{\varepsilon}}}}\,\underline{\overline{E}}$$

$$\underline{D}_x = \underline{\varepsilon}_{xx}\underline{E}_x + \underline{\varepsilon}_{xy}\underline{E}_y + \underline{\varepsilon}_{xz}\underline{E}_z$$

$$\underline{D}_y = \underline{\varepsilon}_{yx}\underline{E}_x + \underline{\varepsilon}_{yy}\underline{E}_y + \underline{\varepsilon}_{yz}\underline{E}_z$$

$$\underline{D}_z = \underline{\varepsilon}_{zx}\underline{E}_x + \underline{\varepsilon}_{zy}\underline{E}_y + \underline{\varepsilon}_{zz}\underline{E}_z$$

$$\text{Let } \underline{\overline{\overline{\varepsilon}}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$



x,y,z are "principal axes"

Note: $\overline{D} \,//\, \overline{E}$ iff $\overline{E} \,//\, \hat{x}, \hat{y}$, or $\hat{z}$ for $\varepsilon_x \neq \varepsilon_y \neq \varepsilon_z$

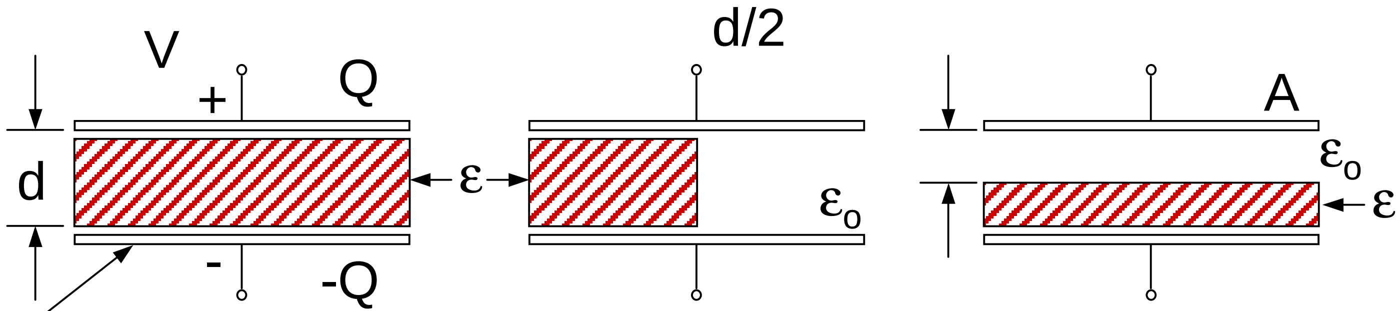
Real $\underline{\overline{\overline{\varepsilon}}}, \underline{\overline{\overline{\mu}}} \Rightarrow$ Lossless medium

HOW TO MAKE ANISOTROPIC MATERIALS

Consider:
$\varepsilon >> \varepsilon_o$

(capacitors)
$Q = CV$

Area A (m^2)



$$C = \frac{\varepsilon_{eff} A}{d}$$

$$\varepsilon_{eff} = \varepsilon$$

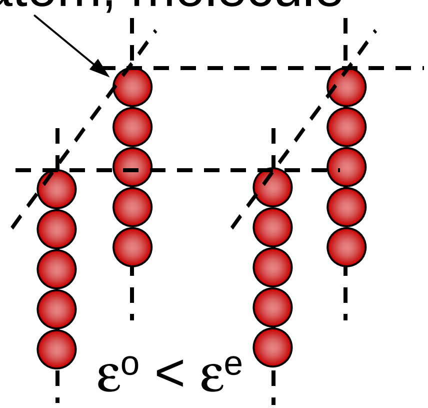
$$C \cong \frac{\varepsilon (A/2)}{d}$$

$$\varepsilon_{eff} \cong \varepsilon/2$$

$$C \cong \frac{\varepsilon_o A}{(d/2)}$$

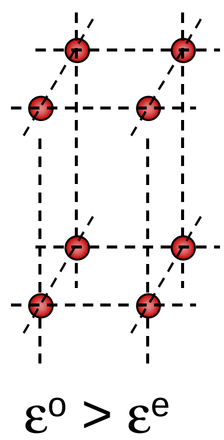
$$\varepsilon_{eff} \cong 2\varepsilon_o$$

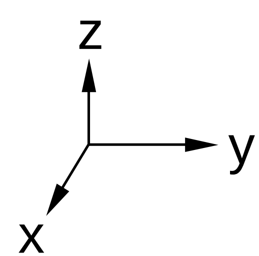
atom, molecule



"uniaxial medium"

$\varepsilon_x = \varepsilon_y = \varepsilon^o$ "ordinary"
$\varepsilon_z = \varepsilon^e$ "extraordinary"





WAVE BEHAVIOR IN UNIAXIAL MEDIUM

Assume wave in $+\hat{z}$ direction, $\sigma = 0$

Derive wave equation:

$$\nabla \times \underline{\overline{E}} = -j\omega\underline{\overline{B}} \qquad \nabla \bullet \underline{\overline{D}} = \rho_f = 0$$

$$\nabla \times \underline{\overline{H}} = j\omega\underline{\overline{D}} \qquad \nabla \bullet \underline{\overline{B}} = 0$$

$$\underline{\overline{D}} = \overline{\overline{\varepsilon}}\,\underline{\overline{E}} \qquad \overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon^e & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}, \quad \overline{\overline{\underline{\mu}}} = \mu$$

Therefore $\nabla \times \left(\nabla \times \underline{\overline{E}}\right) = \nabla\left(\nabla \bullet \underline{\overline{E}}\right) - \nabla^2 \underline{\overline{E}} = -j\omega\mu\nabla \times \overline{H} = \omega^2 \mu \overline{\overline{\varepsilon}}\,\underline{\overline{E}}$

Does $\nabla \bullet \underline{\overline{E}} = 0$ here? Yes, (let's skip proof) can test final solution

Therefore $\nabla^2 \underline{\overline{E}} + \omega^2 \mu \overline{\overline{\varepsilon}}\,\underline{\overline{E}} = 0 \Rightarrow$ 3 equations (x,y,z components)

$$\underbrace{\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right.}_{}\left.+ \frac{\partial^2}{\partial z^2}\right]\left[\hat{x}\underline{E}_x + \hat{y}\underline{E}_y + \hat{z}\underline{E}_z\right] + \omega^2 \mu \overline{\overline{\varepsilon}}\,\underline{\overline{E}} = 0$$

Assume = 0 (UPW in z direction)

This leads to 2 decoupled equations for x and y polarization

BIREFRINGENT MEDIA

Decoupled wave equations:

$$\left[\frac{\partial^2}{\partial z^2} + \underbrace{\omega^2\mu\varepsilon^e}_{\triangleq (k^e)^2}\right]\underline{\underline{E}}_x = 0\,,\quad k^e = \omega\sqrt{\mu\varepsilon^e},$$

(x-pol equation)

$$\left[\frac{\partial^2}{\partial z^2} + \underbrace{\omega^2\mu\varepsilon}_{\triangleq (k^o)^2}\right]\underline{\underline{E}}_y = 0\,,\quad k^o = \omega\sqrt{\mu\varepsilon}$$

(y-pol equation)

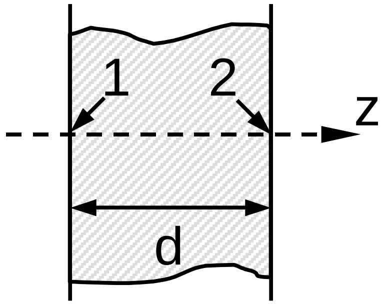
Where $\underline{\underline{E}}_x \propto e^{-jk^e z} = e^{-j\left(\omega/v^e\right)z} \Rightarrow \begin{cases} v^e = 1/\sqrt{\mu\varepsilon^e} \\ v^o = 1/\sqrt{\mu\varepsilon} \end{cases}$

Thus the x- and y-polarized waves propagate independently at different velocities

If $v^e < v^o$ then $v^e \rightarrow$ “slow-axis velocity”

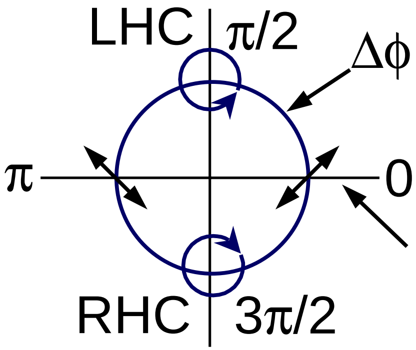
BIREFRINGENT MEDIA

Example:

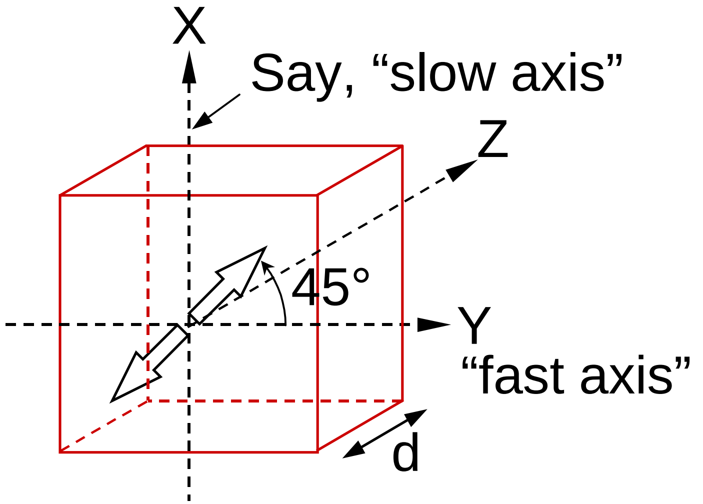


$$\overline{\overline{E}}_1 = E_o(\hat{x}+\hat{y}) \quad 45^\circ \text{ linear pol. input}$$

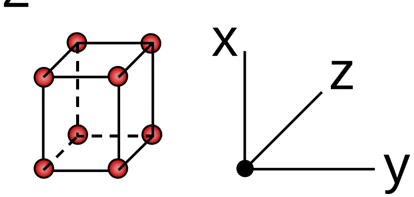
$$\overline{\overline{E}}_2 = E_o\,\underbrace{\hat{x}e^{-j\phi^e} + \hat{y}e^{-j\phi^o}}_{\text{What pol.?}}, \qquad \phi^e = k^e d,\quad \phi^o = k^o d$$

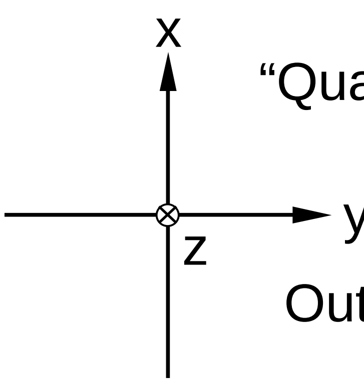


$$\Delta\phi \triangleq \phi^e - \phi^o = (k^e - k^o)d$$



"Quarter wave plate"





Output: $d \ni \Delta\phi = \pi/2$

Demo; Polaroids

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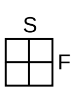
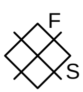
2) ⇕ ⊕ ⇔ ⇒

3) ⇕  ⇔ ⇒

4) ⇕  ⇔ ⇒

5) ⇕ MICA ⇔ ⇒

6) GEARS