

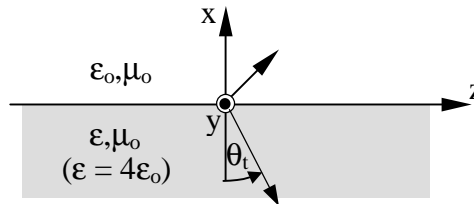
6.013 (New) Electromagnetics and Applications

Problem Set 5	Issued: October 1, 2002 Due in Recitation: October 9, 2002
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Suggested Reading: Text, Sections 4.1, 4.2 < p.132, 3.5, 4.3.

Problem 5.1

Assume a flat semi-infinite slab of permittivity $\epsilon = 4\epsilon_0$ exists for $x < 0$ (ϵ, μ_0), and that free space (ϵ_0, μ_0) is located above ($x > 0$), as illustrated. For each of cases (a) - (b), find the magnitude and direction of the indicated field vector inside the slab, i.e., in the region $x < 0$. Please indicate this direction by some linear combination of the unit vectors \hat{x} , \hat{y} , and \hat{z} .



- a) A uniform static $\vec{E} = (\hat{x} + \hat{z})E_0$ exists wherever $x > 0$; what is \vec{E}_s inside the slab ($x < 0$) and at what angle is it relative to the x axis?
- b) A uniform plane wave at frequency f_0 has $\vec{H} = (\hat{x} + \hat{z})H_0 e^{jk_x x - jk_z z}$ for $x > 0$ and is incident at 45 degrees upon the same slab from the region $x > 0$ ($\theta_i = 45^\circ$); now what is the direction of \vec{H}_s inside the slab? (It depends on θ_t ; we do not need to find the magnitude of \vec{H}_s .) Note that a static magnetic field at a dielectric boundary has the same direction on both sides.
- c) What is the surface charge σ_s for case (a)?

Problem 5.2

Assume that at a particular moment the ionospheric electron density n_e (m^{-3}) increases linearly from zero to 10^{12} between the altitudes of 100 and 200 km, and that n_e above 200 km is irrelevant to this problem.

- a) A certain ionospheric sounding system transmits radio pulses vertically and determines the density profile of the ionosphere by measuring the round-trip delay as a function of radio frequency. From what effective height h_0 is the wave reflected at $f = 5$ MHz?

- b) At 15 MHz, for what range of zenith angles will the radiowave be reflected back toward earth? (Zenith angle is the angle between vertical (zenith) and the upward direction of propagation.) Sketch qualitatively a typical trajectory for such a reflected wave.
- c) Assume we are operating at radio frequencies so high that the ionosphere becomes irrelevant because the refractive index of the atmosphere dominates. Assume the refractive index n_a of the atmosphere at the terrestrial surface is $1 + 10^{-4}$, while that of space is unity (by definition). Now, for what range of zenith angles will the radiowave be reflected back toward earth? (Assume a flat earth and uniform atmosphere.)

Problem 5.3

- a) A uniform plane wave traveling in vacuum is normally incident upon a planar medium (glass) with $\epsilon = 4\epsilon_0$ and $\mu = \mu_0$. What fraction of the incident power is reflected from the front surface? If this were the lense of a camera (which has two surfaces) and if the camera employed several lenses in series (the best cameras do), what sort of problems might this type of scattered light cause?
- b) Automobile headlights are incident upon a wet road on a dark and stormy night at a 10-degree grazing angle. Assume (for purposes of this problem only) that water has a permittivity of $4\epsilon_0$ and that $\mu = \mu_0$. (i) Approximately what fraction of the incident power would be reflected into an on-coming driver's eyes for the TE component of the light? (ii) For the TM component? (iii) Would properly polarized eyeglasses help the on-coming driver? Briefly explain your answers.

Problem 5.4

A digital communications system uses optical pulses propagating along a hypothetical optical fiber for which the propagation delay time is a function of optical frequency ω . Assume k for the wave in the fiber is:

$$k \cong (2\omega/c)(1 - a/\omega)$$

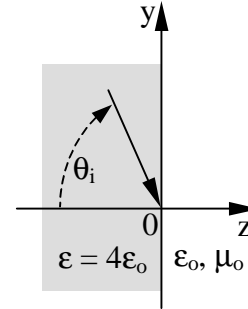
where $a/\omega \cong 10^{-3}$. Approximately what are the phase and group velocities, v_p and v_g , for this fiber? Will the pulses distort (change shape) as they travel down this line? Briefly explain.

Problem 5.5

While swirling her Seven-Up a 6.013 student notices fingerprints where her fingers gently support the glass. Quickly realizing that she is viewing her fingertips beyond the critical angle θ_c determined by the glass and air, she notes that she can measure the depths of the valleys between the tiny ridges that define those fingerprints, since she knows the nominal wavelength λ of visible light is $\sim 0.5 \times 10^{-6}$ meters. While she savors her drink, we can help her by computing:

a) The critical angle θ_c for an interface between air and a highly refractive glass having an index of refraction $n = 1.5$. That is, $\theta_c = ?$

b) The decay length α^{-1} [m] for evanescent waves in air if the wave the glass is incident (or emitted) at an angle $\theta_i = 60^\circ$, which is beyond the critical angle. We define an incident wave at angle θ_i as $\vec{E} = \vec{E}_o e^{+jky-jkz}$ inside the glass, and as $\vec{E}_t e^{+jky-\alpha z}$ in the air outside (between the ridges of fingertips), as suggested by the figure. That is, $\alpha^{-1} = ?$



c) This is one of those rare experiments that can be performed safely at home (assuming Seven-Up or water). Beyond the critical angle the side of the smooth-walled glass appears silvery in the absence of fingerprints, but where the fingertips press closer to the glass than $\sim \alpha^{-1}$, the silvery appearance is replaced locally by the color of the fingertip ridges. Depending on finger pressure, the valleys between the ridges will widen, shrink, or disappear altogether. Based on this experiment, approximately how deep are the ridges on your fingers? Ignore refractive differences between the drink and the glass and assume $n = 1.5$.

