MAGNETIC FORCES ON CHARGES

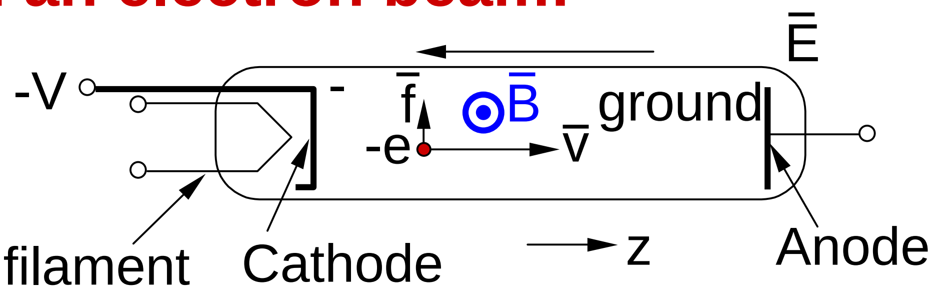
Lorentz Force Law:

$$\bar{f} = q\left(\bar{E} + \bar{v} \times \mu_o \bar{H}\right) \quad \text{Newtons}$$

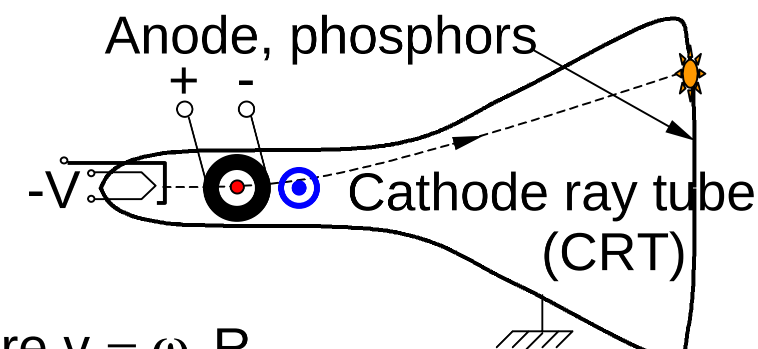
q = electric charge (Coulombs)
$\bar{v}$ = velocity vector (m s^{-1})

Example: magnetic forces on an electron beam:

Lateral forces: $\bar{f} = q\bar{v} \times \mu_o \bar{H}$



Electrostatic deflection best for small V,
magnetic deflection best for large V,v.

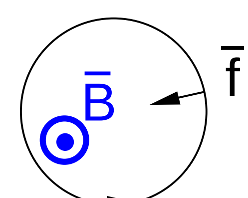


Cyclotron motion:

$$|f| = ev\mu_o H = ma = m_e \omega_e^2 R = m_e v \omega_e, \text{ where } v = \omega_e R$$

$\omega_e = e\mu_o H/m_e$ "Electron cyclotron frequency" $\left[rs^{-1}\right]$

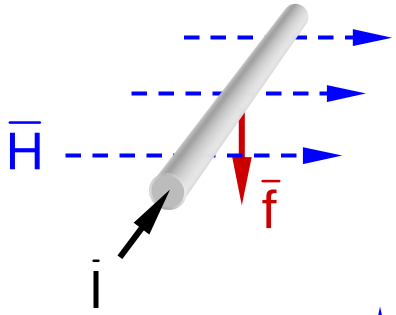
e.g. $\omega = 1.6 \bullet 10^{-19} \times 0.1 / 9.1 \times 10^{-31} \Rightarrow 2.8$ GHz (~MRI)



MAGNETIC FORCES ON CURRENTS IN WIRES

Force equation:

$\bar{f} = q\left(\bar{E} + \bar{v} \times \mu_o \bar{H}\right)$ Newtons

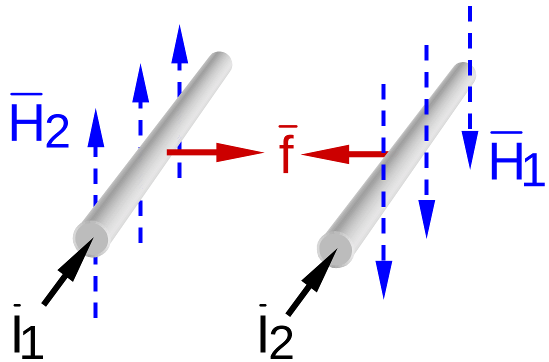


Force on wire [Nm^{-1}]:

$\bar{f} = nq\bar{v} \times \mu_o \bar{H} = \bar{I} \times \mu_o \bar{H}$, where

n is the number of conduction electrons m^{-1}

at $\bar{v}$ and $\bar{I}$ is the current vector $= nq\bar{v}$



Force attracting parallel wires:

$|f| = I\mu_o H = \mu_o I^2 / 2\pi r = 2I^2 \times 10^{-7} / r \ \left[Nm^{-1}\right] \quad \left(\mu_o = 4\pi 10^{-7}\right)$

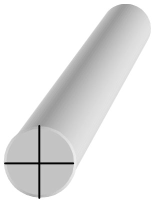
$\int_C \bar{H} \bullet d\bar{s} = I = 2\pi r H \Rightarrow H = I/2\pi r$

Example: $|f| = 2 \times 100^2 \times 10^{-7} / 10^{-2} = 0.2[N]$, attract or repulse

Pinch effect:

This experiment defines μ_o and predicts "pinch effect"

The four quadrants of a wire squeeze together $\propto I^2/r$ and can crush wire, limiting the maximum achievable current density



VOLTAGES PRODUCED BY MOTION ACROSS $\overline{H}$

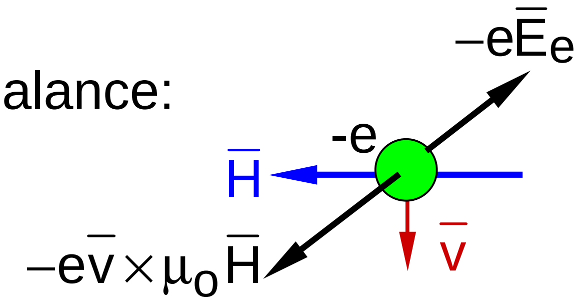
Consider electron inside moving wire:

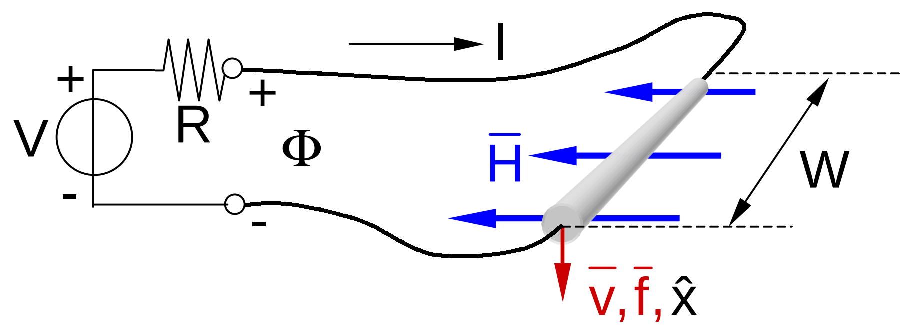
Force on that electron: $\overline{f}_e = -e\left(\overline{E}_e + \overline{v} \times \mu_o \overline{H}\right)$

For open-circuit wire: $\overline{f}_e = -e\left(\overline{E}_e + \overline{v} \times \mu_o \overline{H}\right) = 0 \Rightarrow \overline{E}_e = -\overline{v} \times \mu_o \overline{H}$ inside

Open-circuit voltage across wire: $\Phi = E_e W = v \mu_o H W [V]$, where W = wire length

Force balance:





Electric fields inside conductors:

Total force on free conduction electrons is produced by $\overline{v} \times \overline{H}$ plus an opposing $\overline{E}_e$

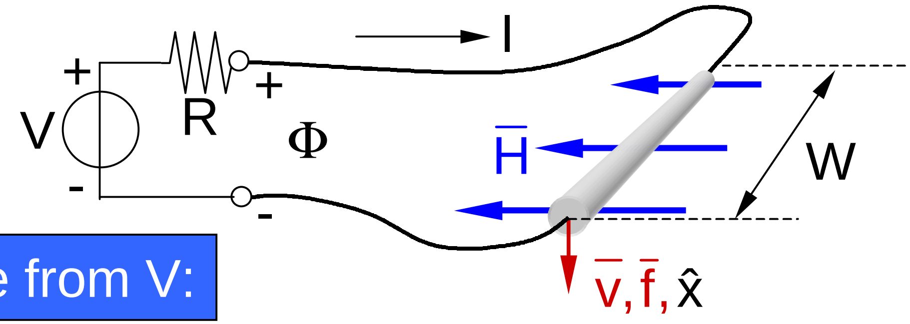
$\overline{E}_e$ is produced by charge distributions that build inside wire until all electrons see $\overline{f} = 0$

MAGNETIC MOTORS ARE ALSO GENERATORS

Current I, force f, and power P produced:

$$I = (V - \Phi)/R$$

$$\bar{f} = \bar{I} \times \mu_o \bar{H} W = \hat{x}\mu_o HW(V - \Phi)/R$$



Mechanical power delivered by wire from V:

$$P_m = \bar{f} \bullet \bar{v} = v\mu_o HW(V - \Phi)/R = \Phi(V - \Phi)/R \, [W]$$

Electrical power delivered to wire by V,R:

$$P_e = VI - I^2R = V(V - \Phi)/R - (V - \Phi)^2/R$$

$$= [(V - \Phi)/R]\,[V - (V - \Phi)] = \Phi(V - \Phi)/R \, [W]$$

Therefore: **Electrical power ⇔ Mechanical power**

It is motor if mechanical power out > 0: i.e. if $V > \Phi = v\mu_o HW$

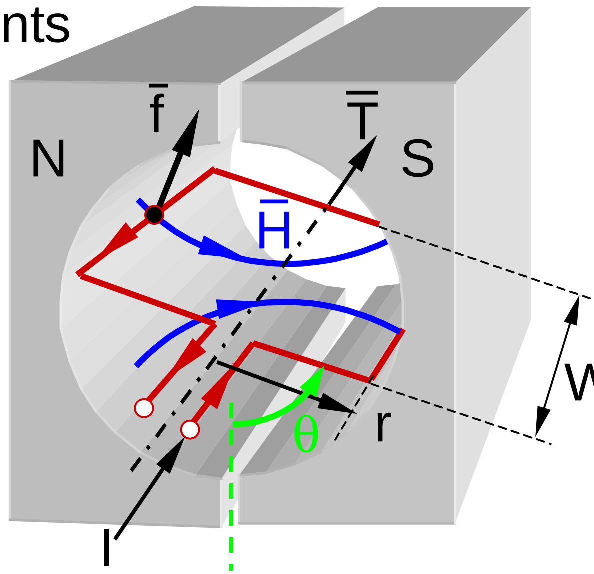
It is generator if electrical power out > 0: i.e. if $V < \Phi$, or $v > V/\mu_o HW$

i.e. when the motor "back voltage" $\Phi > V$.
In unloaded motors $V = \Phi$ and $I = 0$.

ROTARY WIRE MOTOR

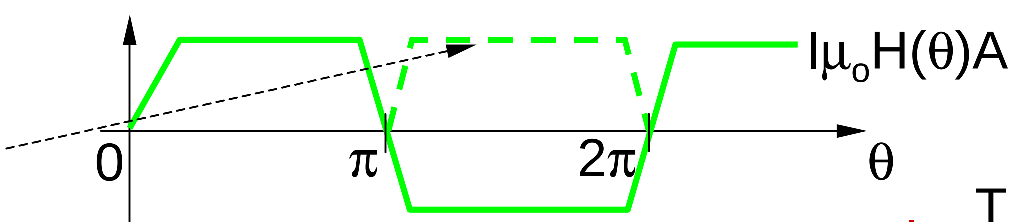
Single wire loop spinning in uniform $\overline{H}$:

Total force (torque) is sum of forces on wire segments
Axial forces from wires at ends cancel
Tangential forces add $\Rightarrow$ torque = 2f[m^{-1}]Wr
f = $I\mu_oH$ (= $NI\mu_oH$ for N-turn coil)
T = $2I\mu_oHWr = I\mu_oHA$ [Nm], A is loop area
Torque is a vector $\overline{T} = \overline{r} \times \overline{f}$



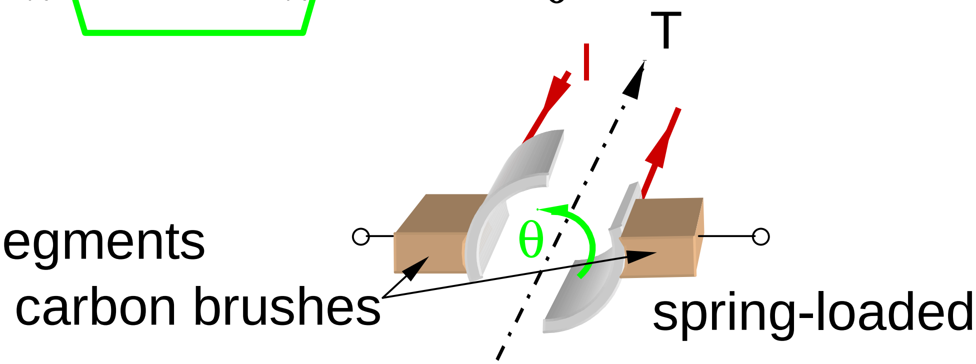
Torque varies with θ:

If I = constant:
With commutator



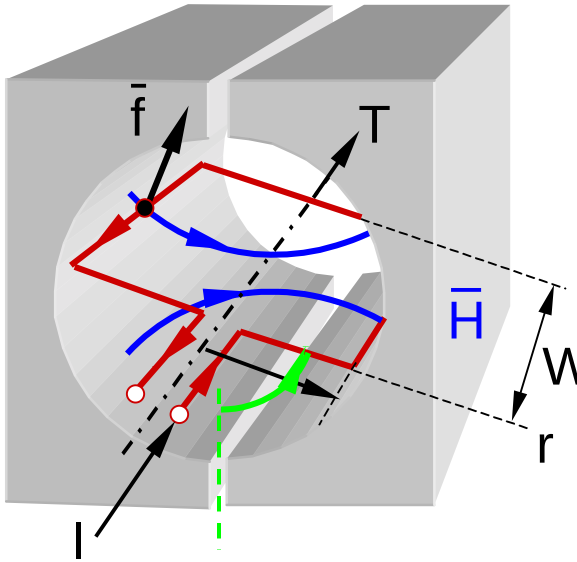
Commutators:

Switch currents to maximize torque
Can have N coils, 2N commutator segments



TWO-POLE (N-S) COMMUTATED MOTOR

Design assumptions:



$\mu_o H \cong$ constant = 1 Tesla (10^4 gauss)
One 100-turn loop (N = 100) of area A = 10^{-3} [m^2]
V = 24 volts, perfectly commutated

Maximum ω, unloaded (T = 0):

ω is angular frequency
Back-voltage Φ = V = 24 [V] in equilibrium
$\Phi = 2NE_eW = 2Nv\mu_oHW = NA\mu_oH\omega = 24$
$\omega = 24/NA\mu_oH = 24/(100 \times 10^{-3}) = 240 \Rightarrow 2292$ rpm
(More typical values for $\bar{B}$ are 0.5 $\Rightarrow$ ~4,600 rpm max)

Maximum torque when ω = 0:

Assume power supply is limited to I = 10 [A]
$T = NI\mu_oHA = 100 \times 10 \times 1 \times 10^{-3} = 1$ [Nm] ($\rightarrow$ 100N at r = 1 cm)

MOTOR TORQUE/POWER/SPEED RELATIONS

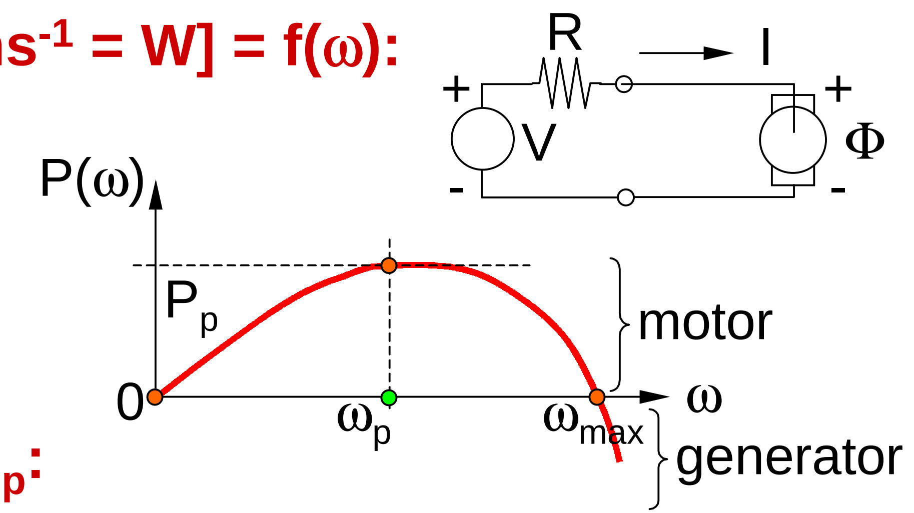
Mechanical power output = $\omega T[Nms^{-1} = W] = f(\omega)$:

$P = \omega T = \omega NI\mu_o HA$

$I = (V - \Phi)/R$

$\Phi = NA\mu_o H\omega$

$P = \omega T = \omega N(V - NA\mu_o H\omega)\mu_o HA/R$



Maximum mechanical power out P_p:

$\partial P/\partial\omega = 0 \;\Rightarrow\; V = 2\omega_p NA\mu_o H$ [so at ω_p we have $\Phi = V/2$] $\Rightarrow$

$\omega_p = V/2NA\mu_o H = \omega_{max}/2$ [$V = NA\mu_o H\omega_{max}$]

$P_p = \omega_p T = (V/2NA\mu_o H)N(V - [NA\mu_o H\, V/2NA\mu_o H])\mu_o HA/R = V^2/4R$

At maximum power, motor is matched load of impedance R!

($\Phi = V/2$ is across motor, V/2 is across R)

Assume $I_{max} = V/R = 192A$ and V = 24 volts. Then R = 0.125 ohms, and

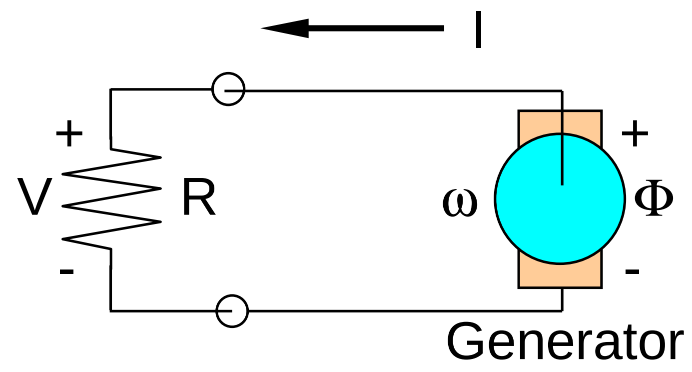
$P_p = 24^2/4\times 0.125 = 1.152$ Kw

Motor design strategy:

To minimize motor weight (cost), boost ωNIH

GENERATOR POWER/FREQUENCY RELATIONS

Electrical power out $P = \omega T = \Phi I$:

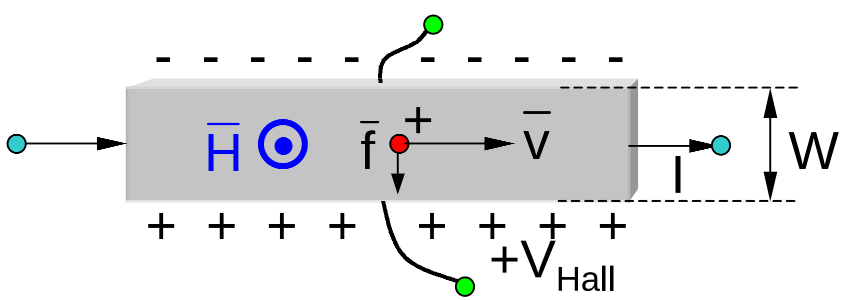


$\Phi = NA\mu_o H\omega$
$I = \Phi/R$
$P = \Phi I = \Phi^2/R = \omega^2(NA\mu_o H)^2/R$

To maximize power out $\Rightarrow P_p$:

ω is limited by vibration, lubricant, rotor fragmentation, and air viscosity (drag); smooth balanced rotors with $\omega r < c_s$ (~300 ms^{-1}) are best. Φ < breakdown voltage. I limited by magnet survival (~200 C), insulation melting, and cooling; heat capacity allows transient peaks.

Hall effect sensors:



$$\overline{E}_e = -\overline{v} \times \mu_o \overline{H} \Rightarrow V_{Hall} = v\mu_o HW$$

$V_{Hall} \Rightarrow$ H if v is known, $\Rightarrow v = I/nq$ if H is known (study carriers)