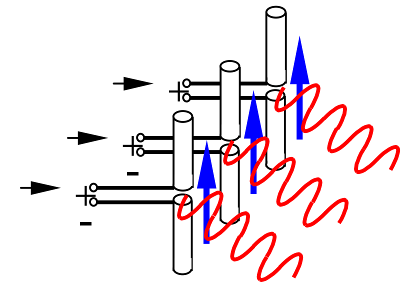
SUPERPOSITION OF PHASORS; ANTENNA ARRAYS/APERTURES

Superposition of Waves, Applications:

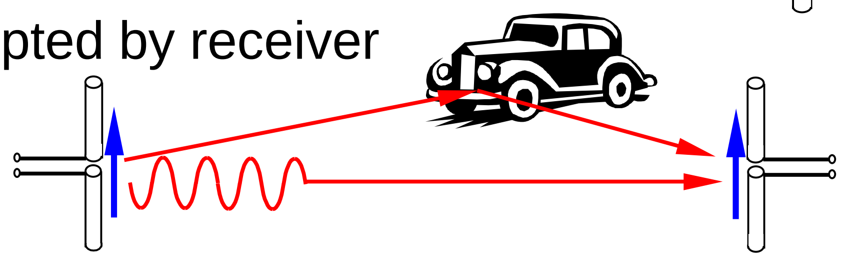


Multiple waves launched by antenna

(Antennas designed to yield desired pattern)

Multiple reflections intercepted by receiver

(Multipath)



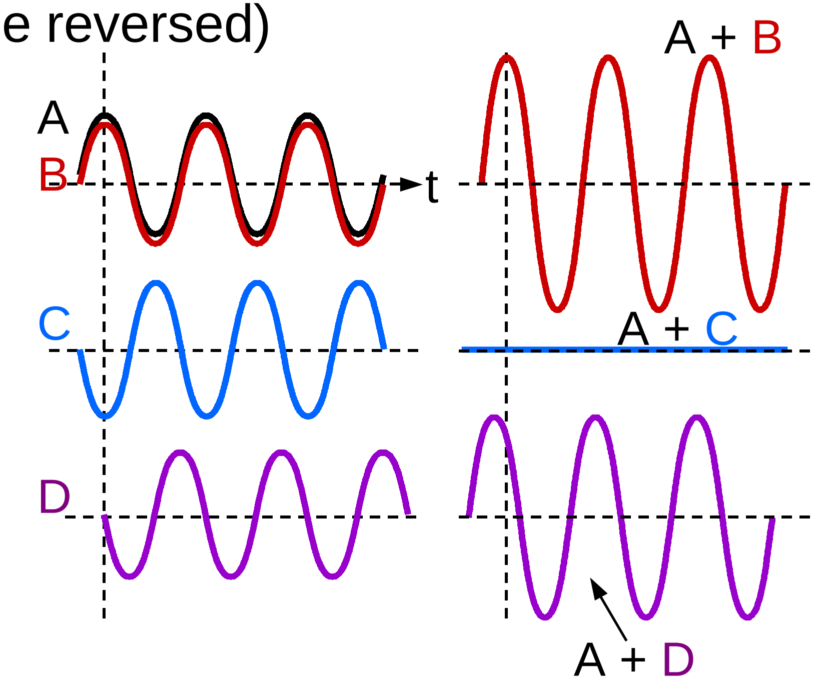
Multiple points of interception in receiver antenna

(Same as transmitting antenna, time reversed)

$\cos \omega t + \cos(\omega t + \phi) = A \cos(\omega t + \theta)$

$[= 2 \cos(\omega t + \phi/2) \cos(\phi/2)]$

Note: fields add, powers do not

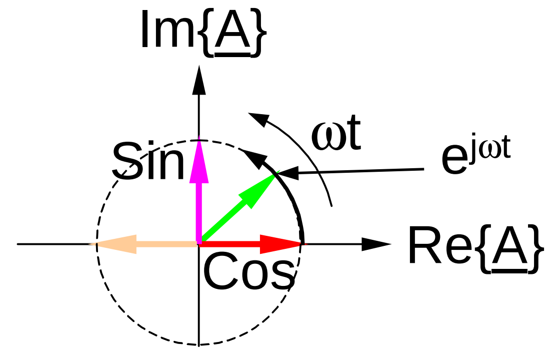


SUPERPOSITION OF PHASORS

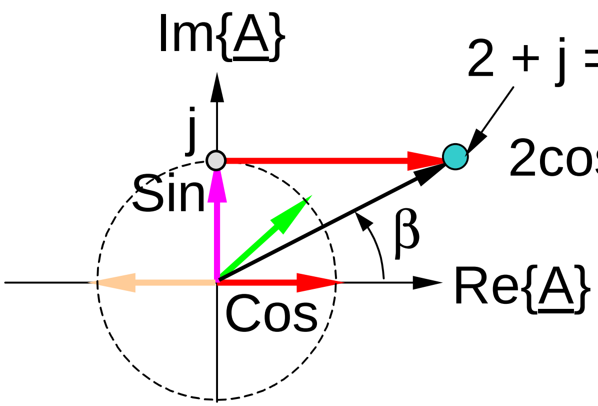
Graphical Method:

$$A(t) = R_e\{\underline{A}e^{j\omega t}\} = R_e\{[R_e\{\underline{A}\} + jI_m\{\underline{A}\}][\cos\omega t + j\sin\omega t]\}$$

$$= R_e\{\underline{A}\}\cos\omega t - Im\{\underline{A}\}\sin\omega t$$



Superposition of Phasors:



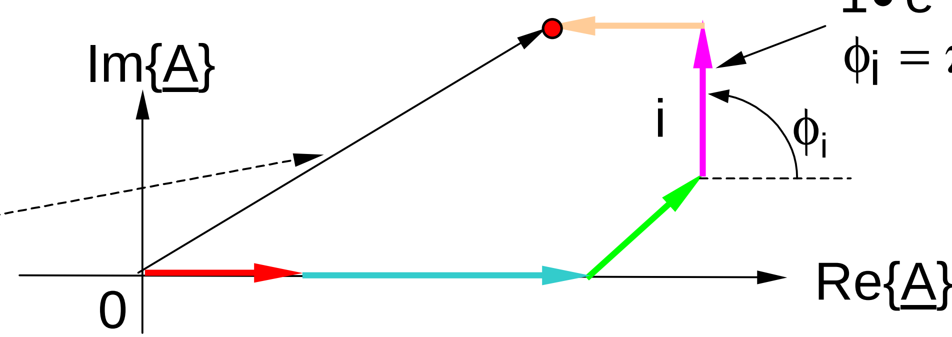
$2 + j = (1^2 + 2^2)^{0.5}\, e^{j\beta} \Rightarrow$

$2\cos\omega t - \sin\omega t = (1^2 + 2^2)^{0.5}\cos(\omega t + \beta)$

("phase lead" of β)

$1 \bullet e^{j\pi/2} \rightarrow |A_i| = 1,$

$\phi_i = \pi/2$



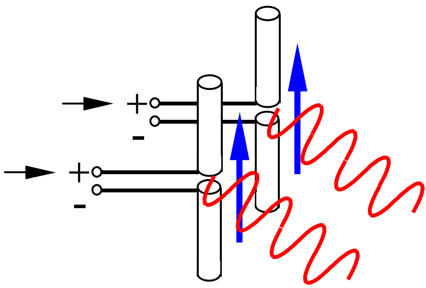
$\underline{B} = \sum_i |\underline{A}_i| e^{j\phi_i}$

ANTENNA ARRAYS

Superposition of Waves:

$$\underline{\bar{E}}(r,\theta,\phi) = \sum_i \underline{a}_i \underline{\bar{E}}_i e^{-jkr_i}$$

$$= \underline{\bar{E}}\left(\sum_i \underline{a}_i e^{-jkr_i}\right) = (\text{element factor } \underline{\bar{E}})(\text{array factor})$$

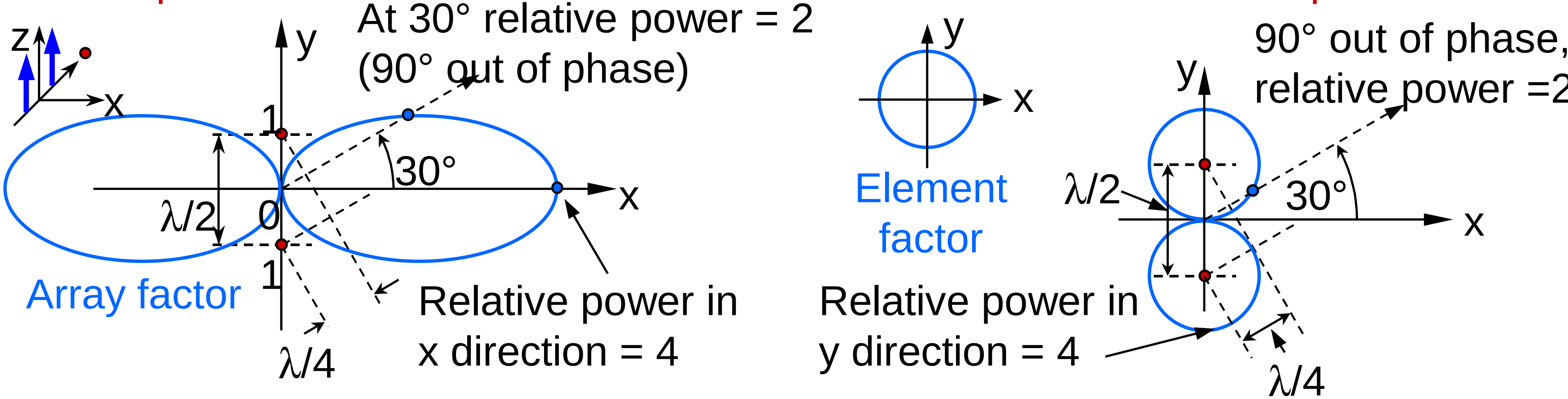


$\underline{\bar{E}}(r,\theta,\phi)$ can be factored if elements have the same orientation and pattern (so $\underline{\bar{E}}_i = \underline{\bar{E}}$), but different locations $\bar{r}_i$, and amplitudes and phases $\underline{a}_i$, where $\underline{a}_i$ characterizes the currents driving each radiating element

Example, horizontal arrays of vertical dipoles:

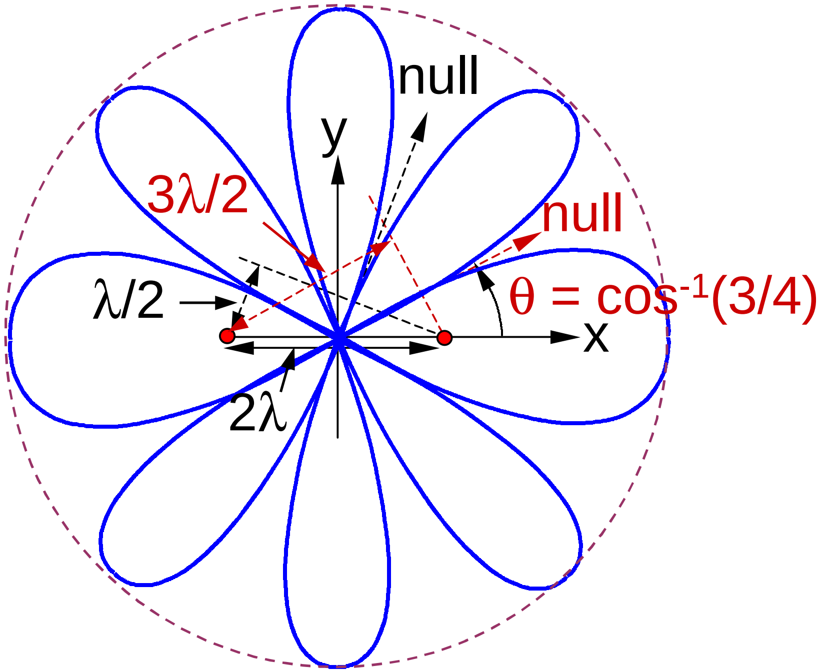
In phase:

• 180° out of phase:



TWO-ELEMENT ARRAYS

In-Phase, 2λ Separation:



Nulls at:

$\theta = \cos^{-1}([\lambda/2]/2\lambda)$

$\theta = \cos^{-1}(3\lambda/2]/2\lambda)$

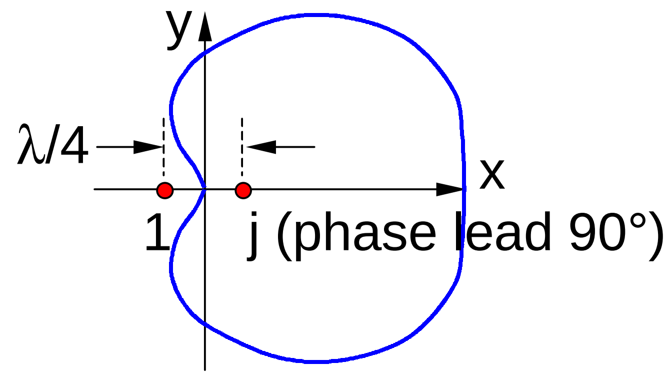
Peaks at:

$\theta = 0$

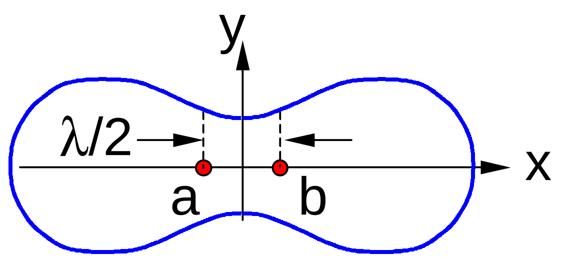
$\theta = \cos^{-1}(\lambda/2\lambda)$

$= \pi/3$

90° phase, λ/4 separation



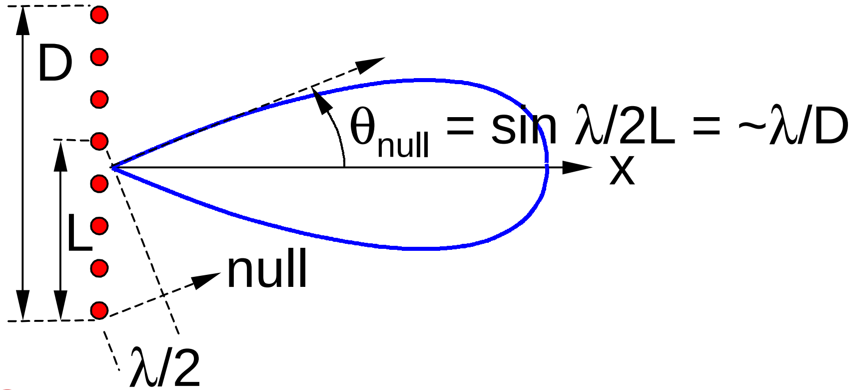
180° phase, λ/2 separation unequal sources



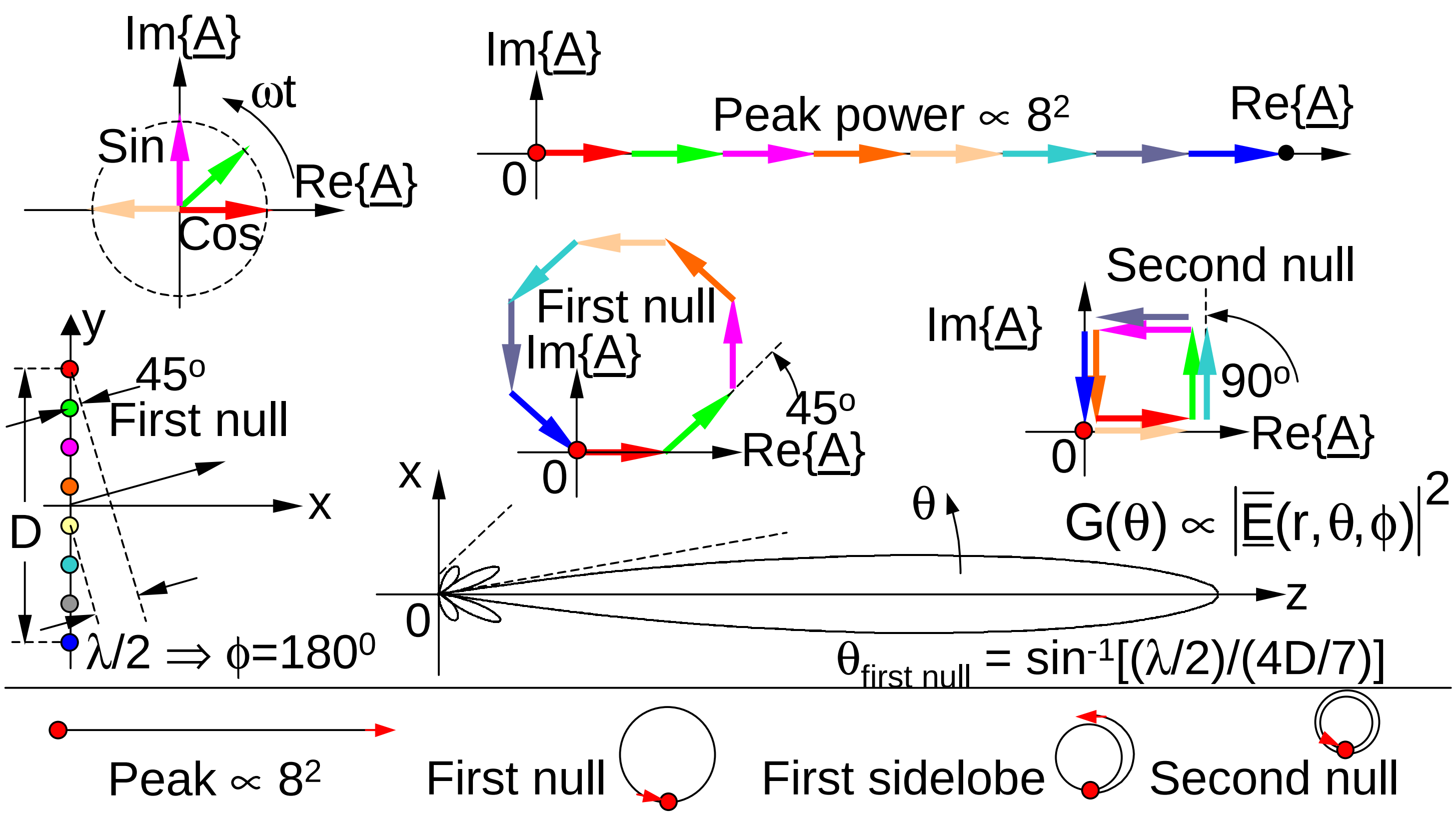
2 unequal sources cannot produce a null

LINEAR ANTENNA ARRAYS

Linear uniformly excited array:



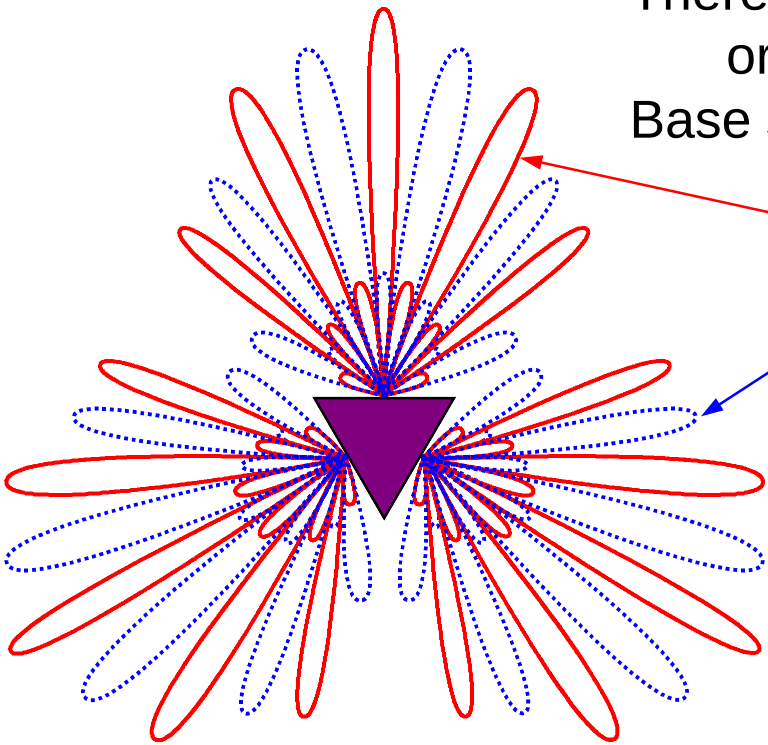
Phasor for Uniform 8-Element Array:



FREQUENCY REUSE

Cell phones:

Frequency allocations are limited
Therefore multiple narrowband signals or other orthogonal signals are used (e.g., TDM)
Base station antennas enable frequency reuse

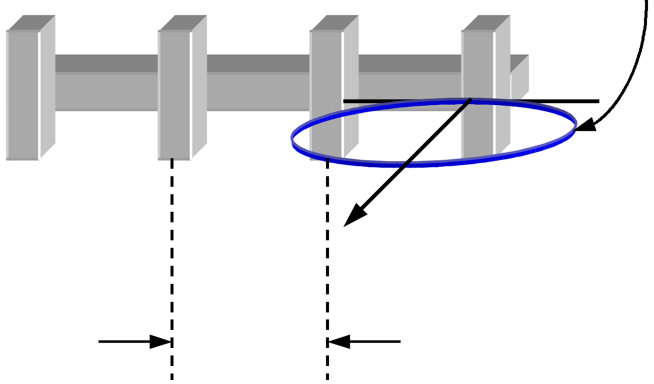


All frequencies, $\alpha = 0$, $\underline{V}_i = \underline{V}_o e^{jm\alpha}$

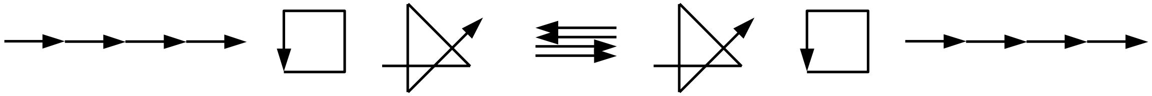
All frequencies, $\alpha = \pi$

Element factor $\overline{\underline{E}}(\theta)$

Typical 4-antenna face



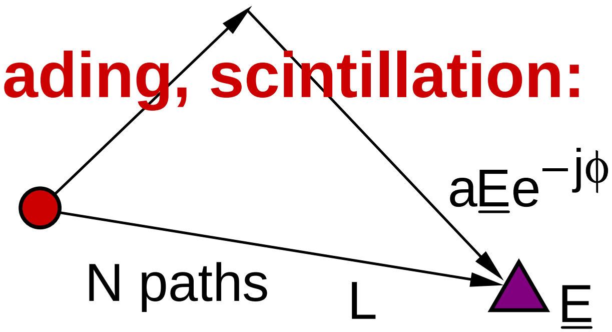
Assume 3λ



Most clients can access non-faded frequencies

MULTIPATH EFFECTS

Fading, scintillation:



Causes of fading:

- Moving reflectors (trucks, trees) vary ϕ
- Moving sources and receivers
- Variations in c due to temperature, humidity
- Polarization rotation or variation

Examples:

FM radios in moving cars click during nulls below FM threshold
Snowy TV broadcast stations waiver as planes fly overhead or trucks pass
Strength of radio stations varies with the weather (also due to refraction)
Ionosphere (faraday) rotates linear polarization $\lesssim$ 3 GHz, causing fades

Doppler shift Δf:

If the path L to the source increases at $v = \partial L/\partial t$, we lose $f_D = (\partial L/\partial t)/\lambda$ cycles per second $= v/\lambda = f_o v/c$ Hz, so $f_D = f_o(1 - v/c)$ Hz

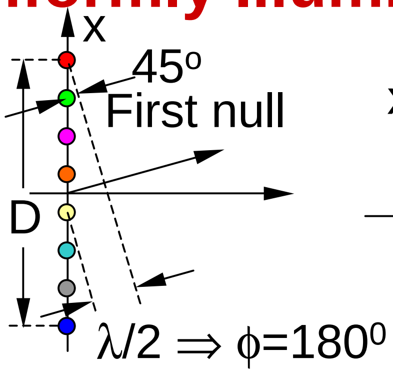
Remedies:

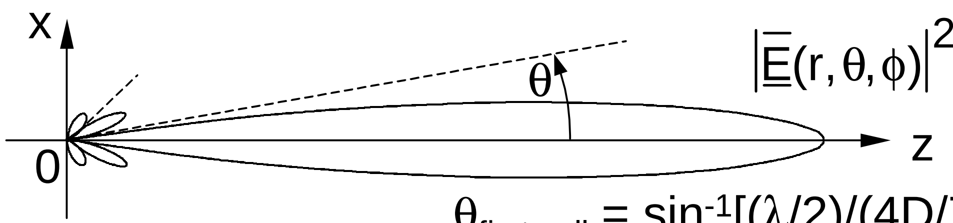
Doppler shift – retune receiver
Fading – high sensitivity and dynamic range; error-correction codes; space, frequency, polarization diversity

DIFFRACTION

Uniformly Illuminated Aperture:

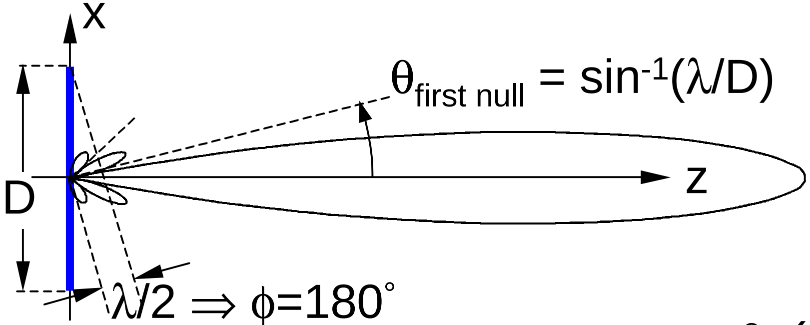
$$\underline{B} = \sum_i |\underline{A}_i| e^{j\phi_i} \rightarrow \int_x \underline{A}(x) e^{j\hat{r}\bullet\bar{x}} dx$$

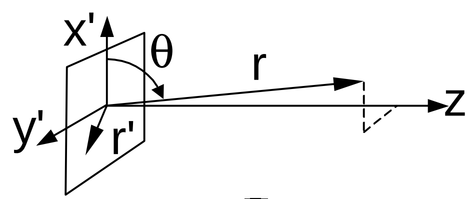


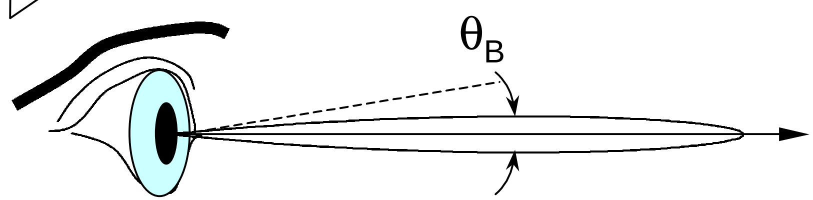


$$\theta_{\text{first null}} = \sin^{-1}[(\lambda/2)/(4D/7)]$$

Planar Aperture:







$$\theta_B(\text{eye}) \cong \sin^{-1}(\lambda/D) \cong \underbrace{0.5\times10^{-6}/10^{-3}}_{5\times10^{-4}\text{ radians}} \cong 1 \text{ arc min}$$

Uniformly Illuminated Aperture A:

$$\bar{\underline{E}}_{ff} \cong \hat{\theta}\left(jke^{-jkr}/2\pi r\right)\cos\theta \int_A \underline{E}_x(x',y')e^{jk\hat{r}\bullet\bar{r}'}da'$$

(~Fourier transform)(r is distance from origin to ff)