6.013(New) Recitation 2: Wireless Radio and Optical Links

A. Review

Wireless radio links were introduced in Lecture 1. The basic equations introduced there are repeated below. First is the equation for the *gain over isotropic* G_t of the transmitting antenna, which is defined as the ratio of: (1) the power P_r [Wm^{-2}] at distance r transmitted in a particular direction θ,ϕ, to (2) the power that would be radiated if the antenna were isotropic (radiating equally in all directions); isotropic power [Wm^{-2}] is the total power transmitted P_T divided by $4\pi r^2$.

$$G_t(\theta,\phi) = P_r(\theta,\phi,r)/(P_T/4\pi r^2) \quad (1)$$

The power received at the other end of the link P_{rec} [W] equals the radiation intensity there $P_r(\theta,\phi,r)$ [Wm^{-2}] times the *antenna effective area* $A_e(\theta,\phi)$ [m^2]:

$$P_{rec} = P_r(\theta,\phi,r)A_e(\theta,\phi) = P_r(\theta,\phi,r)G_{rec}(\theta,\phi)\lambda^2/4\pi \quad (2)$$
$$= G_t(\theta,\phi)(P_T/4\pi r^2)G_{rec}(\theta,\phi)\,\lambda^2/4\pi$$

where we have used the very important relation between the gain and effective area of any antenna [$A=G\lambda^2/4\pi$]. Therefore,

$$P_{rec} = P_T\, G_t\, G_{rec}\, (\lambda/4\pi r)^2 \text{ Watts} \quad (3)$$

The energy received per bit at the receiver E_b [Joules] must be greater than some nominal *sensitivity threshold*, which for most good receivers is:

$$E_b > \sim 4\times 10^{-20} \text{ Joules/bit for typical digital radio receivers} \quad (4)$$

Therefore a link handling data at a rate of M [bits/sec] must receive at least:

$$P_{rec} = M\, E_b \text{ [Watts = J/sec]} \quad (5)$$

Finally, a circuit connected to a lossless antenna, whether it is transmitting or receiving, will see impedance with a real part called the *radiation resistance* R_r [ohms] and a reactive part. The radiation resistance of a transmitting antenna corresponds to the power lost, not as heat, but as radiation as the current i(t) into it transmits P_T Watts. Simple power considerations yield:

$$R_r = P_T/< i^2(t) > \quad (6)$$

B. Example of a Wireless Link for Interstellar Communications

Since distance clearly limits the ease of communications, let's consider an ambitious goal—communications with another stellar system, perhaps one light year away, so $r = ct \cong 3\times10^8 \times 3\times10^7 = 9\times10^{15}$ meters. Powerful antennas today have gain $\cong 10^7$ or more, and we can assume we have one such antenna on each end of the link. We can further assume $\lambda = 0.1$ [m] and that M = 1 bit/sec. The necessary received power is $ME_b \cong 1 \times 4\times10^{-20}$ [W]. Using (3) we can now calculate the required radiated power P_R:

$$P_R = P_{rec}/[G_tG_{rec}(\lambda/4\pi r)^2] \cong 4\times10^{-20}/[10^{14}(0.1/4\pi\ 9\times10^{15})^2] = 512\ [W] \qquad (7)$$

This required transmitter power is so low that we can easily achieve higher data rates by boosting that power. If we increase it to a nominal maximum of 1.3 MWatt, the link could then support 2.4 kbps, enough to handle a compressed voice link in "real time" (plus a one-year delay each way). Thus distance is not necessarily a barrier to wireless communications.

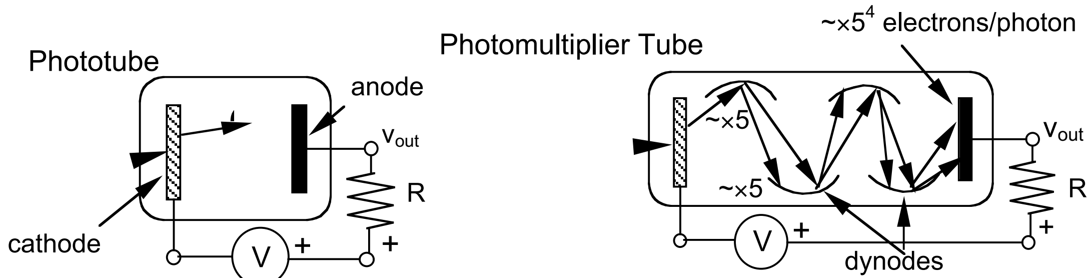
C. Photonic Links

Optical links are increasingly being used for long distance high-data-rate communications because the antenna gains can be so very high. Before analyzing an optical interstellar link, let's review the basic principles of optical detection. First we recall that radio waves and light are governed by exactly the same Maxwell's equations, and both are also comprised of *photons*. The energy E [Joules] of a single photon is:

$$\boxed{E = hf\ \ [J]} \qquad (8)$$

where *Planck's constant* $h \cong 6.625\times10^{-34}$ and f is frequency [Hz]. Therefore the energy of an optical photon is roughly 7 orders of magnitude greater than for a radio photon, so we generally always have many photons in radio systems. Even at the given nominal radio threshold of sensitivity, 4×10^{-20} [J/bit] (see (4)), we require thousands of photons, e.g., at 1 GHz we need $E_b/hf \cong 6\times10^4$ photons. This compares to 5-50 photons required per bit for communications at visible wavelengths, provided that we are not limited by optical interference like sunlight or detector noise. Fewer optical photons are required because we can often detect single-photon arrivals, and we need only enough photons to ensure that they are not due to interference. Note that the optical threshold of sensitivity [J/bit] is several orders of magnitude larger than for the best radio systems, so optical systems are less energy efficient in that sense. Their superiority arises in cases where light beams can be much more highly focused, or can be conveyed at much lower losses through optical fibers.

Typical photon detectors include phototubes and semiconductors. *Phototubes* detect photons having $hf > \Phi$ using the *photoelectric effect*, where Φ is the *work function* [J] of the metal surface (*cathode*) that intercepts the photons. Since the work function of most metals is ~2-6 electron volts (one *electron volt* is e = 1.602×10^{-19} Joules)[1], phototubes do not work well for infrared or longer wavelengths. Higher energy photons, however, can eject an electron from the cathode with a *quantum efficiency* η (probability) on the order of 10-30 percent. These electrons are then pulled in vacuum toward the positively charged *anode* as they contribute current through the *load resistor* R, as illustrated in Figure R1-4. The total current is proportional to the number of incident photons per second.



Yields current pulses larger than thermal noise

Since the thermal noise from the resistor typically exceeds the signal due to a single photon, *photomultiplier tubes* are commonly used for detecting very low light levels, one photon at a time. Their operation is suggested in the Figure above, where the photoelectrons from the cathode are accelerated electrically toward a nearby positively charged *dynode*, which they impact with sufficient energy (say 50-100 volts) that they dislodge perhaps five new electrons. Each of these electrons is then accelerated toward the second dynode at a still higher voltage where each again dislodges another ~5 electrons. A typical photomultiplier tube with n dynodes might then have a *gain* of 5^n, where n might typically be 7-12. Although only a fraction η of the incident photons produce an original electron, this becomes, for example, an avalanche of $\sim5^n \cong 10^4\text{-}10^7$ electrons at the anode. Such current pulses are sufficient to overwhelm most noise sources, so each detected photon can then be counted individually. Electrons emitted spontaneously by the cathode, due perhaps to cosmic rays or thermal effects, then constitute the dominant noise; spontaneous (noise) electrons from the dynodes are amplified less and can be distinguished and ignored.

The collecting areas of such tubes can be enhanced with lenses or mirrors having cross-sections of A [m^2], where the effective area A_e of such telescopes approximates A and, as before:

$$\boxed{A_e = G_o\lambda^2/4\pi \ \ [m^2]} \tag{9}$$

Phototubes are generally large (several cubic inches), expensive, and fragile, and therefore *semiconductor photodiodes* are more commonly used. Semiconductors have the additional advantage that they can be made to respond better to infrared wavelengths. Figure A below illustrates the *energy diagram* for a typical *p-n junction*, where the

[1] Note that the energy associated with charge Q moving through potential V is QV Joules, so QV = 1 e.v. = e×1 = 1.602×10^{-19} Joules.

vertical axis is electron energy E and the horizontal axis is distance z perpendicular to the planar junction. The lower cross-hatched area is the *valence band* and the upper area is

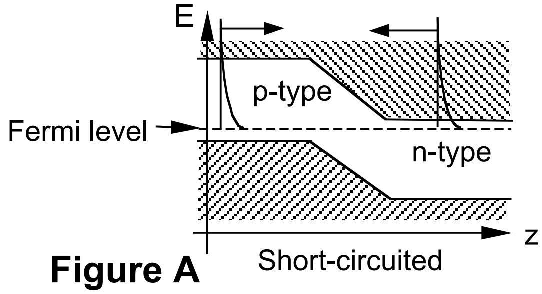


Figure A

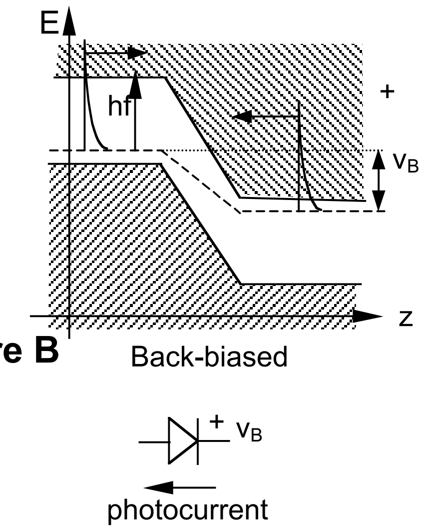


Figure B

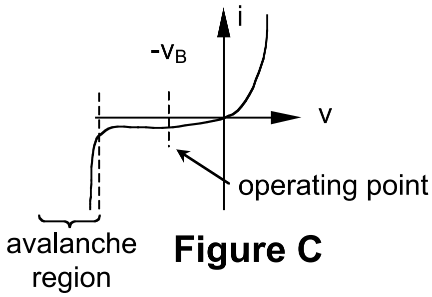


Figure C

the *conduction band*. They are separated by the *band gap*, which is ~1.12 electron volts for silicon, and ranges from 0.16 for InSb (Indium Antimonide) to ~7.5 for BN (Boron Nitride), depending on the semiconductor.

Electrons can move freely if they have been excited into the conduction band, but not if they remain in the valence band. Photons with energy greater than the bandgap can excite electrons from the valence band into the conduction band to enhance device conductivity. In semiconductors the *Fermi level* is that energy level which characterizes the maximum energy of abundant electrons available for excitation into the conduction band. It is analogous to sea level. Impurities in the semiconductors create electron donor or acceptor sites that easily release or hold, respectively, electrons, and these sites determine the Fermi level, which sits just above the valence band for p-type semiconductors and just below the conduction band for n-type semiconductors, as illustrated

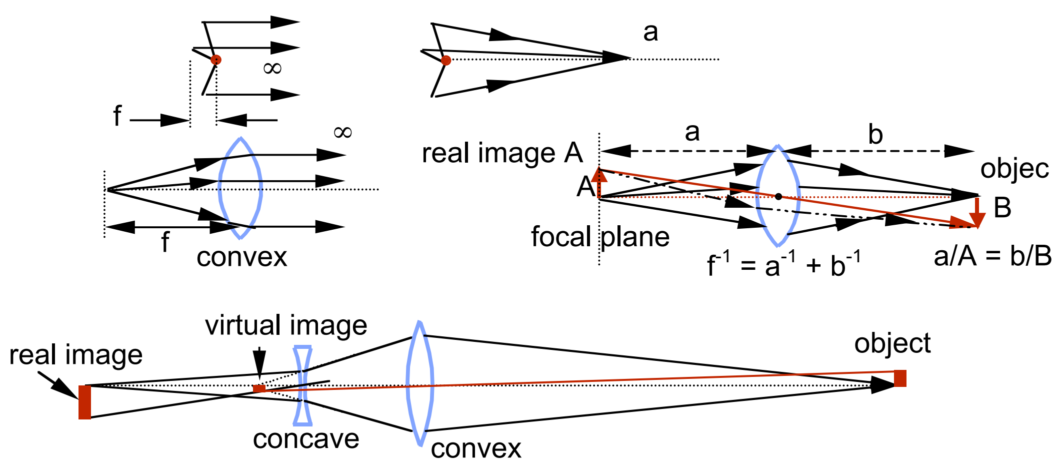
If the p-n junction is short-circuited externally, the Fermi level must be the same on both halves. The random thermal excitation of electrons produces an exponential (*Boltzmann*) distribution in energy, as suggested in the figures, the upper tails of which are in the conduction band on both halves of the junction. When the device is short-circuited, these current flows from excitations in the p and n halves of the junction must balance, so the external current is zero. If, however, the diode is back-biased by V_B volts as illustrated in Figure B, then the two exponential tails do not balance and a net back-current current flows, as suggested in Figure C, which is the I-V characteristic for a p-n junction. The back current approaches an asymptote equal to the integrated number of electron excitations per second into the conduction band for the p-semiconductor. When the junction is forward biased, the current increases roughly exponentially.

Such p-n junctions operate as photodiodes when they are back-biased, for any photon penetrating the p-semiconductor near the junction can excite an electron into the conduction band so it can move to the n-semiconductor and out into the external circuit. The quantum efficiencies of good semiconductor diodes are often above 90 percent, but in small band gap (infrared) semiconductors there are substantial excitations by heat alone. Therefore such diodes may be cooled or are used with light sources sufficiently intense that photoelectrons dominate the current flow.

If photodiodes are sufficiently back-biased, they can enter the *avalanche region* where the excited electron is accelerated sufficiently as it moves through the semiconductor that it can excite another electron into the conduction band; these two can now accelerate and excite even more, exponentially, until they have all lost sufficient energy that further excitations are not possible. In response to single detected photons such *avalanche photodiodes* (*APD*'s) can produce output pulses of $\sim 10^4$ electrons that stand out sufficiently above the noise that the photons can again be counted individually. Note that the number of photons per second is proportional to input power, and therefore to the square of the incident electric field strength.

D. Optical Systems

The principal function of optical systems is to concentrate or focus energy from a source onto a detector, or from a source onto a distant target. Typical optical systems are illustrated in Figure R1-6, starting with parabolic mirrors that have an infinite focal length f, or that focus rays at a finite distance a.



Convex lenses intercepting parallel rays also generally focus them at some *focal length* f. If we have an object B located at distance b from a convex lens, then light rays from that source are focused on A, forming an *image* at distance a, where $f^{-1} = a^{-1} + b^{-1}$. Also, if object B and image A have heights B and A, respectively, then a/A = b/B, so we can have magnification (if a > b) or demagnification (if a < b). Note that a ray from B which passes straight through the center of the (thin) lens is generally not bent, which explains why a/A = b/B.

Systems can have multiple lenses. For example, the illustrated combination of a convex and *concave lens* intercepts rays from a distant object on the right so as to produce a *real image* on the left. Conversely, an object on the left will produce a real image on the right, and also a *virtual image*, which is the apparent source of rays emanating to the right from the convex lens.

Diffraction-limited optical systems have angular resolution (or beamwidth) θ_B that is no better than $\sim\lambda/D$ radians, where λ is wavelength and D is the aperture diameter [m]. In general, the better the angular resolution, the higher the corresponding gain G, where we can show later that $G \cong 4\pi/\theta_B^2$.

E. Examples of Optical Communications Links

Section B examined a system for interstellar communications over a one-light-year link using 10-cm wavelength. Let's now assume we want instead to communicate with the planet Mars (say $r \cong 10^{11}$ meters) using a very modest 1-watt 0.5-micron wavelength laser attached to a 10-cm telescope. Such a telescope has a diffraction limit of ~1 arc second (60 arc minutes per degree, 60 arc seconds per arc minute) which approximates the limits imposed by the terrestrial atmosphere. Let's further assume we are using a 1-meter telescope on Mars (atmospheric seeing does not limit telescope size on Mars because the atmosphere is so thin) together with an APD or photo-multiplier tube that can extract an average of one bit of information per 10 photons (or 10hf Joules = E_b). We can find the maximum data rate M(bps):

$$M = P_{rec}/E_b = G_t(P_R/4\pi r^2)G_{rec}\lambda^2/(4\pi\times 10hf) \quad (10)$$

Where $G_t = 4\pi A_t/\lambda^2 = (\pi D_t/\lambda)^2$ and where $A_r = 100A_t$. Therefore:

$$\begin{aligned} M &= 100(\pi D_t/\lambda)^4(P_R/4\pi r^2)\lambda^2/(4\pi\times 10hf) \\ &= 100(0.1\pi/[5\times 10^{-7}])^4(1/\pi 10^{22})(5\times 10^{-7})^2/(4\pi\times 10\times 6.625\times 10^{-34}\times[3\times 10^8/5\times 10^{-7}]) \\ &= 620 \text{ kbps} \end{aligned} \quad (11)$$

where we used $f = c/\lambda$, and this data rate is the same in both directions. The propagation delay is $r/c = 10^{11}/(3\times 10^8) \cong 5.5$ minutes. This could support hundreds of compressed telephone channels or excellent internet access.

Suppose instead we wanted to send data among computers and peripherals within a single room ($r \cong 10$ m) with one-milliwatt photodiode emitters (P_R) with nearly isotropic ($G = 1 \cong [\pi D_t/\lambda]^2$) diffraction-limited transmitters and receivers. In this case (11) becomes:

$$\begin{aligned} M &= (P_R/4\pi r^2)\lambda^2/(4\pi\times 10hf) \\ &= (10^{-3}/4\pi 10^2)(5\times 10^{-7})^2/(4\pi 10\times 6.625\times 10^{-34}\times[3\times 10^8/5\times 10^{-7}]) \\ &= 0.004 \text{ bits per second!} \end{aligned} \quad (12)$$

The reason it is easier to reach Mars than across the room is that we assumed the antennas were diffraction limited and isotropic (G=1), implying that their effective areas

A were extremely small. Recall that $A = G\lambda^2/4\pi = \lambda^2/4\pi = 25\times10^{-14}/4\pi$. In practice we would instead use ~omni-directional receiving photodiodes of perhaps 1-cm diameter that would intercept many more photons ($\times\sim10^{10}$) incoherently. Even though the noise and interference levels would be higher with such a detector, the data rates could then be in the desired 1-Mbps range.