6.013-ELECTROMAGNETICS AND APPLICATIONS

General Content:

Electromagnetics and Applications

- Maxwell's equations and solutions; statics and dynamics, wave phenomena
- Applications: wireless, circuits, media, forces and generators, computer speed, microwaves, optical communications, acoustics, etc.

Mathematical Methods

- Partial differential equations, difference equations, phasors, vector calculus

Problem Solving Techniques

- Perturbation methods, boundary value problems, energy methods, duality

Academic Review

- Mechanics, quantum phenomena, circuits, devices signals, linear systems

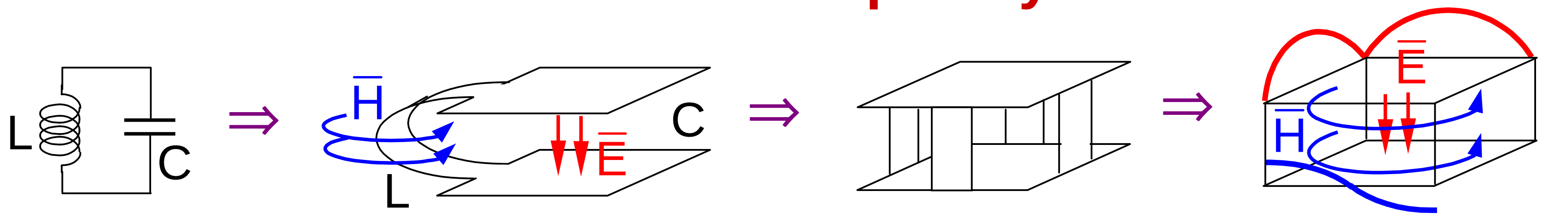
Capstone Subject—Professional Preparation

Follow-on Subjects:

Electromagnetic waves: 6.632
Quasistatics: 6.641

PERTURBATIONS OF RESONATORS

Perturbation of LC Resonator Frequency:



TE$_{101}$ mode

$\omega_o = (LC)^{-0.5}$

If C increases, ω_o decreases ⇒ indenting top center of cavity is equivalent

Calculation of Δf [Hz] due to Shape Perturbations:

Energy method: Assume closed resonator, no energy escape

Therefore total energy $w_T = nhf$, n = #photons at f[Hz]

n = constant, for slow changes in resonator shape

$\Delta f = \Delta w_T/nh$ [Hz]

Assume: EM energy increases Δw_T (±) as work (mechanical) is done on the fields when the walls are forced into their new shape

$\Delta w_T = -f_{orce}\,\Delta z$ [J] (mechanical work done **on** EM f_{orce})

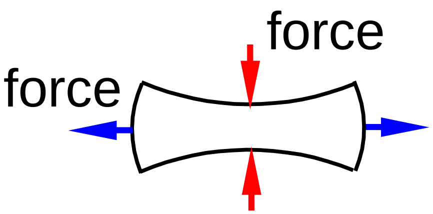
Approach: Find force density F[Nm^{-2}], then Δw_T, then Δf

PERTURBATIONS OF RESONATORS (2)

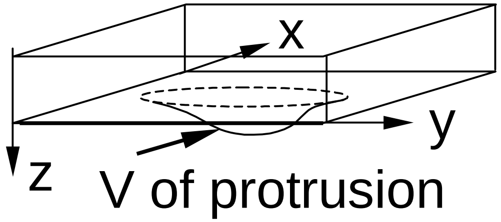
Electromagnetic Forces on Conducting Walls:

Recall: $F_{magnetic} = \mu_o |\overline{H}|^2/4 \, [Nm^{-2}][Jm^{-3}] = W_m$ Repulsive force density

$F_{electric} = \varepsilon_o |\overline{E}|^2/4 = W_e$ Attractive force density



Electromagnetic Energy Increase is Δw_T due to Wall Movement:



$$-\Delta w_T = \int dz \int F_{EM}(x,y)dxdy \text{ (work done \textbf{on} moving walls)}$$

$$= \int_V F(x,y)dv = \int_V (W_m - W_e)dv \ [J]$$

$$-\Delta w_T = \Delta(w_m - w_e) \text{ [Joules in V added by change]}$$

V is the volume enclosed by the original and deformed surface contours

V is sufficiently small that $\overline{E}$ and $\overline{H}$ are constant within it

Calculation of Δf [Hz]:

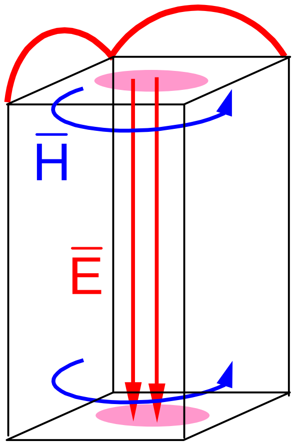
Recall: $\Delta f = \Delta w_T/nh = f(\Delta w_T)/w_T$ [Hz] (where $w_T = nhf$)

Therefore: $\Delta f/f = \Delta(w_e - w_m)/w_T$ where w_m and w_e are values within V

Note: Δw is the increase in w in protrusion; $w \leq 0$ in an indentation

PERTURBATIONS OF RESONATORS (3)

Example—Cavity Resonator (Recall $\Delta f/f = \Delta(w_e - w_m)/w_T$):



Question: Where does indentation lower frequency f_0?

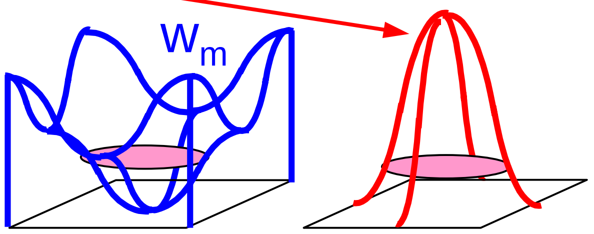
Answer: Within contours 

where Δw_e is negative (removed)

and $\Delta(w_e - w_m)$ is also negative

Note: $w_e \propto \sin^2(\pi x/a)\sin^2(\pi y/a)$

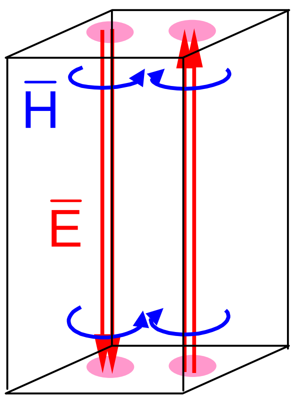
$w_m \propto \cos^2(\pi x/a)\cos^2(\pi y/a)$



Question: Where does indentation raise frequency f_0?

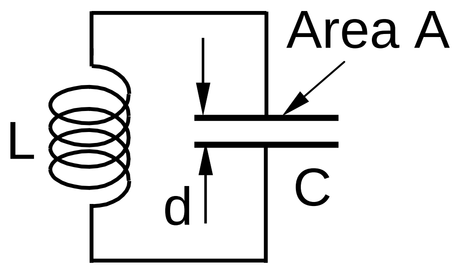
Answer: Everywhere else (for this mode, TM_{101})

Question: Same as above, for TEM_{210} mode?



PERTURBATIONS OF RESONATORS (4)

Example—RLC Resonator:



Problem: If $d \rightarrow d + \Delta d$, then $f \rightarrow \Delta f$; what is Δf [Hz]?

Recall: $\Delta w_e = \left(\varepsilon |\underline{E}|^2 / 4\right)(A\ \Delta d)$ if d increased by Δd

(all else remaining constant) $(w_T = w_e + w_m = 2w_e)$

Therefore: $\Delta f / f_o = \Delta\left(w_e - w_m\right)/w_T = \left(\varepsilon |\underline{E}|^2 / 4\right)(A\ \Delta d) \Big/ \left(2\varepsilon |\underline{E}|^2 / 4\right)(Ad)$

$= \Delta d / 2d$

Check Result:

Recall: $\omega_o = (LC)^{-0.5} = [L\varepsilon A/d]^{-0.5} \rightarrow \omega_o' = (LC')^{-0.5} = [L\varepsilon A/(d + \Delta d)]^{-0.5}$

$C = \varepsilon A/d \rightarrow C' = \varepsilon A/d'$

Therefore: $\omega_o' = [L\varepsilon A/d(1 + \Delta d/d)]^{-0.5} \cong [L\varepsilon A/d]^{-0.5}(1 - \Delta d/2d)$

$\Delta\omega = \omega_o - \omega' \cong \omega_o\ \Delta d/2d$

$\Delta f/f_o = \Delta\omega/\omega_o = \Delta d/2d$, agrees with the above.

Note: The energy perturbation method is approximate; here $\Delta d/d << 1$

HUMAN ACOUSTIC RESONATORS

Human Vocal Tract:

$$f_1 = \omega_1/2\pi = c_s/4d$$
$$= 340/(4 \times 0.16)$$
$$= 531 \text{ Hz}$$

Higher Resonances:

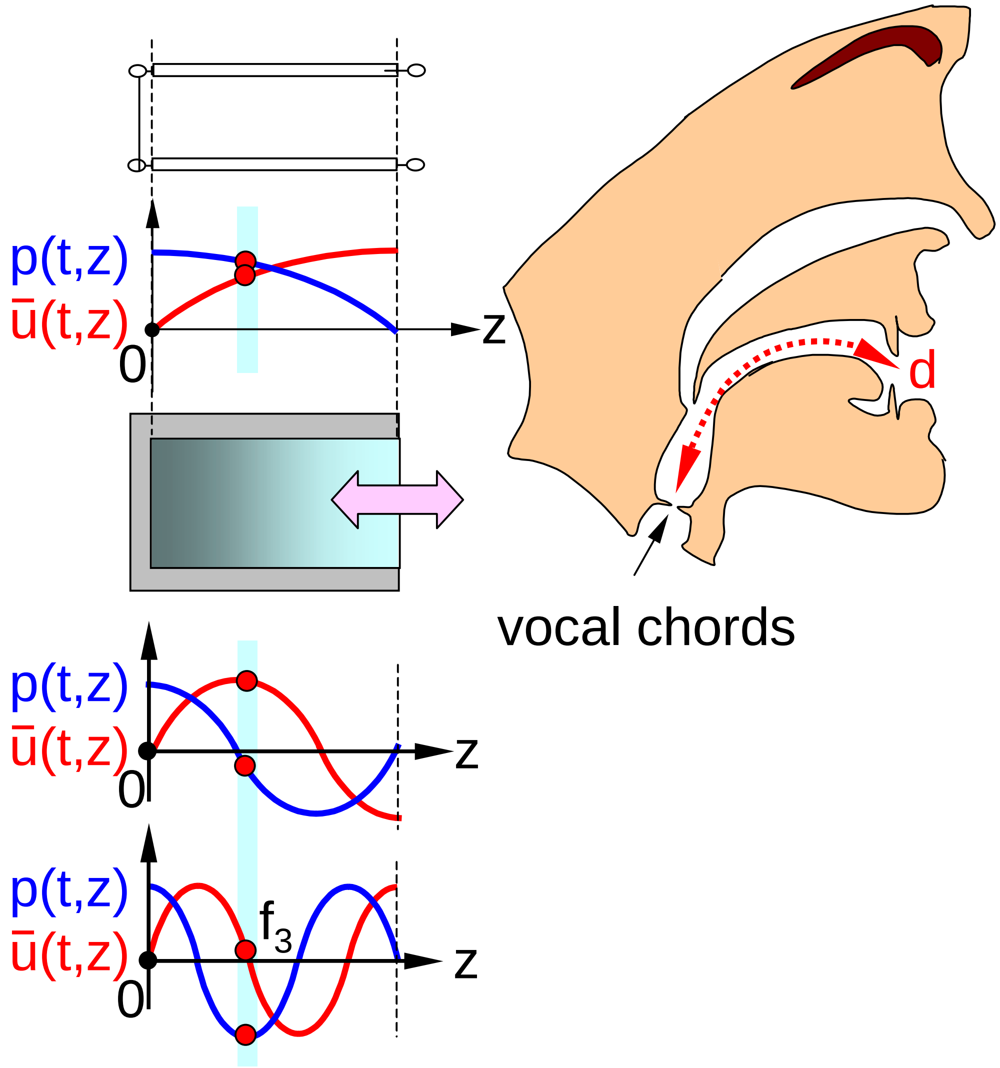
$$f_2 = 3f_1 = 1594 \text{ Hz}$$
$$f_3 = 5f_1 = 2655 \text{ Hz}$$

Energy Densities at Location “ ”

At f_1: $w_p \cong w_u$

At f_2: $w_u >> w_p$

At f_3: $w_p >> w_u$



RESONANCE PERTURBATIONS IN HUMAN VOICES

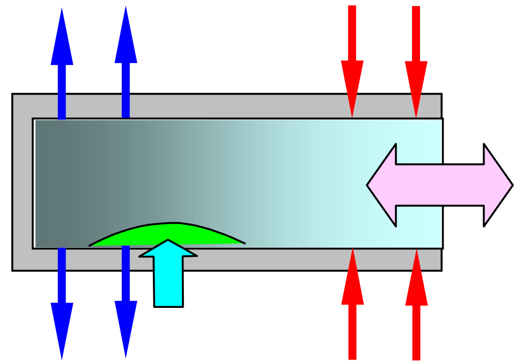
Human Vocal Tract:

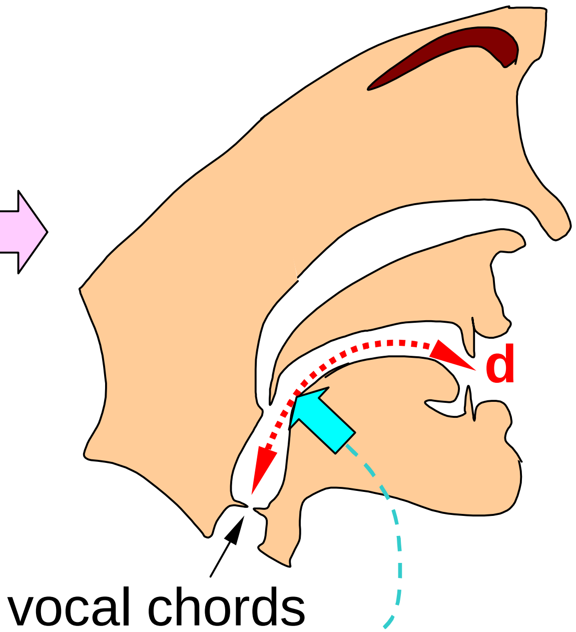
Average forces:

Outward at maximum $|p|$

Inward at maximum $|u|$

(Bernoulli force)





Resonator Total Energy $w_T = \sum^{n}$ phonons:

$w_T = nhf_o = w_p + w_u$ (potential + kinetic energy)

If resonator shape presses inward at p maximum then both w_T and f_o increase

Resonance Perturbations:

$\Delta f/f = \Delta(w_p - w_u)/w_T$

$w_p \cong w_u$ at f_1, $w_u >> w_p$ at f_2, $w_p >> w_u$ at f_3

$f_o \propto c_s \propto (\gamma P_o/\rho_o)^{0.5}$