6.013 Recitation 24:
Dispersion and Interferometry

Pulse spreading due to dispersion

As mentioned previously, one of the key properties of an optical fiber is its dispersion. The phase velocity of a single frequency propagating along the fiber is simply given by $v_p = \omega/k = (\varepsilon\mu)^{-1/2}$. But, since ε can vary with frequency, the phase velocity is generally a function of frequency. Then, as we have seen before, the envelope of a signal (necessarily comprised of a band of frequencies) travels with the group velocity $v_g = d\omega/dk$. If, in addition, the group velocity also varies with frequency, different parts of the signal will travel at different group velocities. The signal will spread and distort. This kind of dispersion is referred to as group velocity dispersion, or GVD.

We can make a simple estimate of how rapidly a pulse spreads in the presence of GVD. Consider a pulse of duration τ_p. It must necessarily (by Fourier analysis) be comprised of a frequency spectrum with a width of at least $\Delta\omega = 1/\tau_p$. As the pulse propagates it spreads, as illustrated in Figure 1, to a width of Δt at a distance of L.

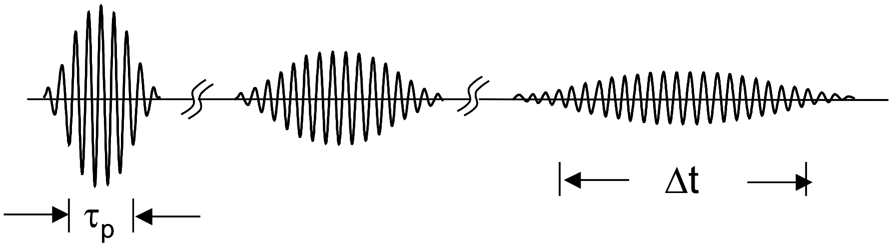

Figure 1: Pulse spreading with distance caused by group velocity dispersion (GVD)

This amount of spreading occurs because the frequency components ω_1 and ω_2 separated by $\Delta\omega$ become separated by

$$\Delta t = \frac{L}{v_{g1}} - \frac{L}{v_{g2}} = L \cdot \frac{d}{d\omega}\left(\frac{1}{v_g}\right) \cdot \Delta\omega = L \cdot \frac{d^2k}{d\omega^2} \cdot \Delta\omega \qquad (1)$$

We have used $(1/v_g) = dk/d\omega$ and assumed a small change in v_g between ω_1 and ω_2. So, the spreading is directly proportional to $d^2k/d\omega^2$ and to the distance L. It is also apparent, since $\Delta\omega = 1/\tau_p$, that the shorter the initial pulse duration τ_p is , the faster it spreads.

Unlike in metallic waveguides or other strongly guiding structures, the dispersion of optical fibers is rather strongly influenced by the dispersion of the material itself. Figure 2 shows how the GVD of optical fiber varies with wavelength. The units of $d^2k/d\omega^2$ are, as per convention, ps^2/km. (A picosecond is 10^{-12} sec.)

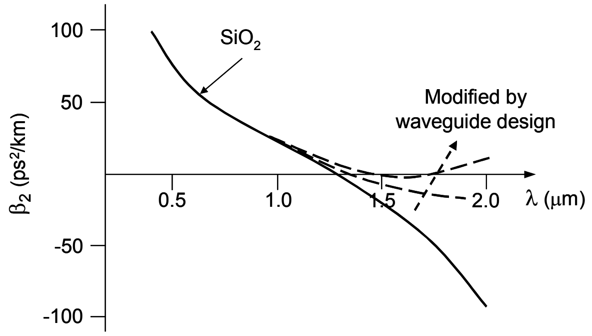

Figure 2: The group velocity dispersion of optical fiber vs. wavelength

The solid line is the GVD for the fused silica (SiO_2) glass material by itself. It has a value of zero at a wavelength of about 1.3 μm, which was therefore initially thought to be the optimum wavelength for optical communications. With the advent of erbium-doped fiber amplifiers (EDFA's) that operate at wavelengths near 1.55 μm, attention moved to those longer wavelengths. Fiber designers found too that they could shift the zero GVD point into the 1.55 μm regime by proper design of the waveguide (generally by making the core smaller and more tightly guiding. It is even possible to make a length of fiber with one sign of GVD to cancel out the GVD of another fiber with the opposite sign. (In fiber with positive $d^2k/d\omega^2$, lower frequencies travel faster; and in fiber with negative $d^2k/d\omega^2$, higher frequencies travel faster.)

Example: Standard single mode optical fiber has a dispersion of 20 ps^2/km at 1.55 μm. If 10ps pulses are used to transmit data at a rate of 40Gb/s, the pulses will broaden to 25ps and begin to interfere strongly with each other after a distance of $L = (25ps)(10ps)/20ps^2/km = 12.5$ km. This is considerably less than the usual 50 km between amplifiers and/or the total span in main trunk lines of 500 km. So, dispersion compensation is necessary.

More generally: The reason for describing GVD in terms of $d^2k/d\omega^2$ is best understood by considering the expansion of the propagation constant k in a Taylor series around a particular center frequency ω_o.

$$k(\omega) = k_o + \frac{dk}{d\omega}(\omega - \omega_o) + \frac{d^2k}{d\omega^2} \cdot \frac{1}{2}(\omega - \omega_o)^2 + ... \qquad (2)$$

Since propagation of the wave is determined by $e^{-jk(\omega)z}$, it becomes clear that the first derivative $\frac{dk}{d\omega} = \frac{1}{v_g}$ is the coefficient of the jkz phase term that varies linearly with frequency, and that the second derivative $\frac{d^2k}{d\omega^2} = \frac{d}{d\omega}(\frac{1}{v_g})$, the GVD term, is related to the jkz phase that varies quadratically with frequency.

From this general description of dispersion in terms of the different derivatives of k(ω) it follows that there is an easy prescription for describing the propagation and spreading of an arbitrary temporal signal given at z=0 by E(t,0). One simply finds its Fourier transform E(ω), multiplies E(ω) by $\exp j[k_o+(dk/d\omega)(\omega) + (1/2)(d^2k/d\omega^2)(\omega^2)]L$, and then takes the inverse Fourier transform to determine E(t,L).

Interferometry

A large variety of measurement, signal analysis, modulation and switching applications rely upon the use of wave interference. This is especially true at optical frequencies because, on the one hand, the short wavelengths provide high resolution and, on the other hand, optical detection systems cannot directly observe small changes in the electric field. Perhaps the simplest, and certainly the most famous, device for utilizing wave interference is the **Michelson interferometer** illustrated in Figure 3.

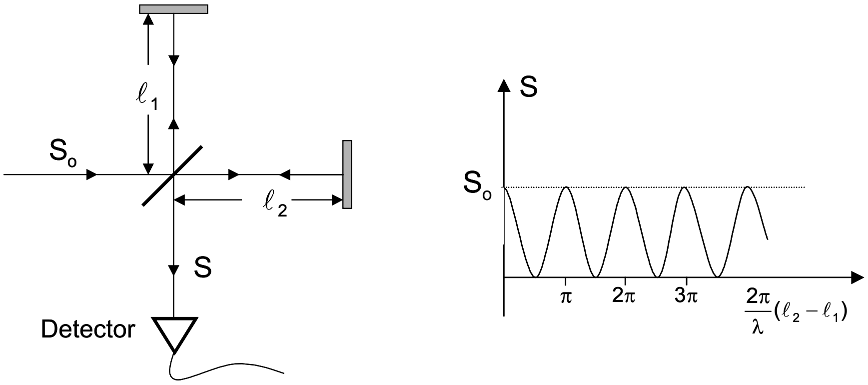

Figure 3: Michelson Interferometer Detected output intensity

The interferometer operation can be idealized as follows. A uniform plane wave $E_o\, e^{-jkz}$ is incident from the left onto a beam splitter that has a reflection coefficient r and a transmission coefficient t . Power conservation requires that $1-|r|^2 = |t|^2$. Both reflected and transmitted beams are subsequently reflected from perfectly reflecting mirrors and return to the beam splitter after traveling distances of $2\,\ell_1$ and $2\,\ell_2$ respectively. Then after being again transmitted and reflected respectively by the beamsplitter they combine (in the downward direction in the figure) to produce a field

$$\begin{aligned} E(x) &= E_o \cdot r \cdot t \cdot (e^{-jk(x+2\ell_1)} + e^{-jk(x+2\ell_2)}) \\ &= E_o \cdot r \cdot t \cdot e^{-jkx} \cdot e^{-jk(\ell_2+\ell_1)} \cdot (e^{jk(\ell_2-\ell_1)} + e^{-jk(\ell_2-\ell_1)}) \end{aligned} \tag{3}$$

The intensity S traveling in the x (downward) direction to the detector is then

$$S = \frac{1}{2\eta}|E(x)|^2 = \frac{1}{2\eta}E_o{}^2 \cdot |r|^2|t|^2 \cdot 4\cos^2 k(\ell_2 - \ell_1) \tag{4}$$

If the beam splitter divides the power 50:50, $|r|^2 = |t|^2 = \frac{1}{2}$ and

$$S = \frac{1}{2\eta}|E(x)|^2 = \frac{1}{2\eta}E_o{}^2 \cdot \cos^2 k(\ell_2 - \ell_1) = S_o \cos^2 k(\ell_2 - \ell_1)$$
$$= S_o \frac{1}{2}\{1 + \cos 2k(\ell_2 - \ell_1)\} \tag{5}$$

So, periodically when $\ell_2 - \ell_1 = m\frac{\lambda}{2}$ $(\text{for } m = 0,1,2...)$ all of the incident intensity S_o reaches the detector. (Where does the power go when the output is zero, i.e. at $\cos^2 k(\ell_2 - \ell_1) = 0$?)

Some of the most accurate distance measurements possible are made with Michelson interferometers by monitoring the output intensity S as one of the mirrors is moved. An optical wavelength is only about 0.5 μm, and it is easy to resolve the change in intensity due to a length change equal to a small fraction of a wavelength. The amplitudes of periodic motions in particular (because they are easier to distinguish from noise) can be resolved with accuracies on the order of the size of an atom. Interferometers are used to determine shifts in the earth's crust, and a major research effort is currently underway to use a interferometer to detect motion due to gravitational waves from outer space.

The Michelson interferometer is also quite often used as a "wavemeter," a tool for determining optical wavelengths and frequencies. A known wavelength is passed through the interferometer for calibration and other wavelengths are identified from their different periodicities in the output. More generally - since the electric field at the output of the interferometer is the coherent sum of the cosinusoidal patterns produced by all the individual incident frequencies, a plot of the output obtained by moving one of the mirrors is in fact the Fourier Transform of the input frequency spectrum. The technique for studying spectra this way (because it was first used for infrared wavelengths where other forms of spectroscopy are less efficient) is called FTIR (Fourier Transform Infrared) spectroscopy.

An alternative device for interferometry is the **Mach-Zehnder interferometer** illustrated in Figure 4.

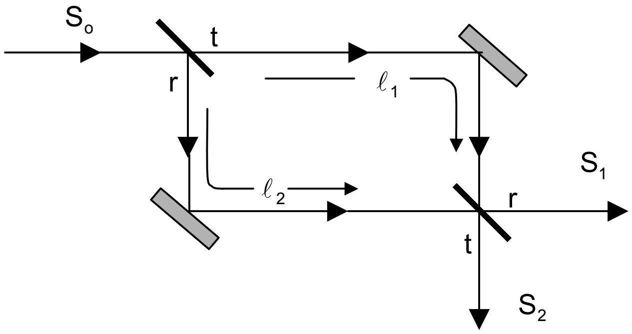

Figure 4: Mach-Zehnder interferometer

In this case the output beam that has intensity proportional to $|r|^2 \cdot |t|^2$ (like the output of the Michelson) emerges to the right. Since the path lengths in this case are ℓ_1 and ℓ_2 rather that $2\ell_1$ and $2\ell_2$ the output is proportional to $\cos^2 \frac{k}{2}(\ell_2 - \ell_1)$. As $(\ell_2 - \ell_1)$ changes, however, intensity that doesn't emerge to the right emerges in the downward direction (rather than being reflected by the interferometer as is the case with the Michelson). By power conservation the intensity of the second output is proportional to

$1 - \cos^2 \frac{k}{2}(\ell_2 - \ell_1) = \sin^2 \frac{k}{2}(\ell_2 - \ell_1)$. Thus,

$$S_1 = S_o \frac{1}{2}(1 + \cos k(\ell_2 - \ell_1) \quad \text{and} \quad S_2 = S_o \frac{1}{2}(1 - \cos k(\ell_2 - \ell_1) \qquad (6)$$

The Mach-Zehnder interferometer has achieved greatly increased importance with the development of integrated optical waveguide technology. It can be implemented simply in the form shown Figure 5. Incoming waves are divided equally into the two waveguide arms of the interferometer. If they combine in-phase again at the second waveguide merger, all of the input power is transmitted. If they combine out-of-phase at the second waveguide merger, there is no output S_1. (In a closed waveguide system the power would then be reflected back out the input port, but in an integrated low-index-contrast dielectric waveguide device the out-of-phase combination produces power in a forward traveling mode that is not trapped by the output waveguide and that therefore radiates away.)

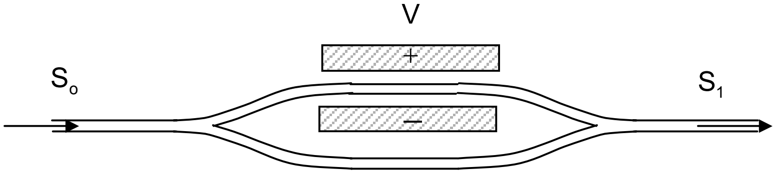

Figure 5: The waveguide Mach-Zehnder (with modulation electrodes)

There are two important applications of the waveguide Mach-Zehnder: (1) as a wavelength filter, and (2) as a modulator, used for putting data on optical beams. Consider first the **filter** application. It follows from the wavelength dependence of the output when $\ell_2 \neq \ell_1$. Assume that the output (S_1) is a maximum at λ_o when $\ell_2 - \ell_1 = \Delta L$, i.e. $\pi\Delta L/\lambda_o = m\pi$. Then transmission is zero for $\lambda = \lambda_o + \Delta\lambda$ when

$$\frac{\pi \Delta L}{\lambda_o + \Delta\lambda} = m\pi + \frac{\pi}{2}$$

Now, assuming that ΔL is large enough so that $\Delta\lambda << \lambda_o$, we obtain

$$\frac{\pi\Delta L}{\lambda_o}\left(1-\frac{\Delta\lambda}{\lambda_o}\right)=m\pi+\frac{\pi}{2} \qquad \Rightarrow \qquad \Delta\lambda=\frac{\lambda_o}{2\Delta L}\cdot\lambda_o \tag{7}$$

Thus, the fractional bandwidth of the filter ($\Delta\lambda/\lambda_o$) is inversely proportional to ΔL.

The second, **modulator**, application is achieved by forming the waveguides in an electro-optic material - a material whose ε changes linearly with applied electric field. Then, a voltage applied between electrodes around one of the interferometer arms (as illustrated in Fig. 5) produces a change in dielectric constant that creates a different effective path length (different phase velocity) in that arm – proportional to the voltage V. A DC bias voltage V_o can be applied in addition to the signal voltage V(t) so that the optical output can be made to either increase or decrease with signal voltage V(t) depending upon the bias. In a typical device made of LiNbO3 $v_p = 1/\sqrt{\varepsilon\mu_o}$ the change in index of refraction $\Delta n = \Delta\sqrt{\varepsilon/\varepsilon_o} \cong 3\cdot 10^{-10}\cdot E$ where E (v/m) is the electric field in the waveguide arm. Assuming that the voltage v is applied across a dimension of 20 μm, then the induced optical phase delay in the arm is

$$\Delta\phi=\frac{2\pi}{\lambda}(1.5\cdot 10^{-5})\cdot\ell\cdot V$$

For a λ = 1.5 μm, the desired $\Delta\phi = \pi$ for maximum optical output change requires a voltage length product 2.6×10^{-2} volt-m. To achieve switching with a voltage of 5 volts then requires the length of the waveguide arm to be about 1 cm.

Example: One might also ask how high a modulation speed is possible. At the speed of light in LiNbO3 ($v_g \cong c/2.2$) it takes about $\Delta t = 7.3 \times 10^{-11}$sec for light to travel beneath the electrode. This transit time limits the modulation bandwidth to frequencies for which $\Delta t < T/2 = 1/2f$, i.e. to $f < 6.8$ GHz. Higher modulation speeds can still be achieved by using traveling wave electrodes so that the modulating voltage can move along with the optical wave.