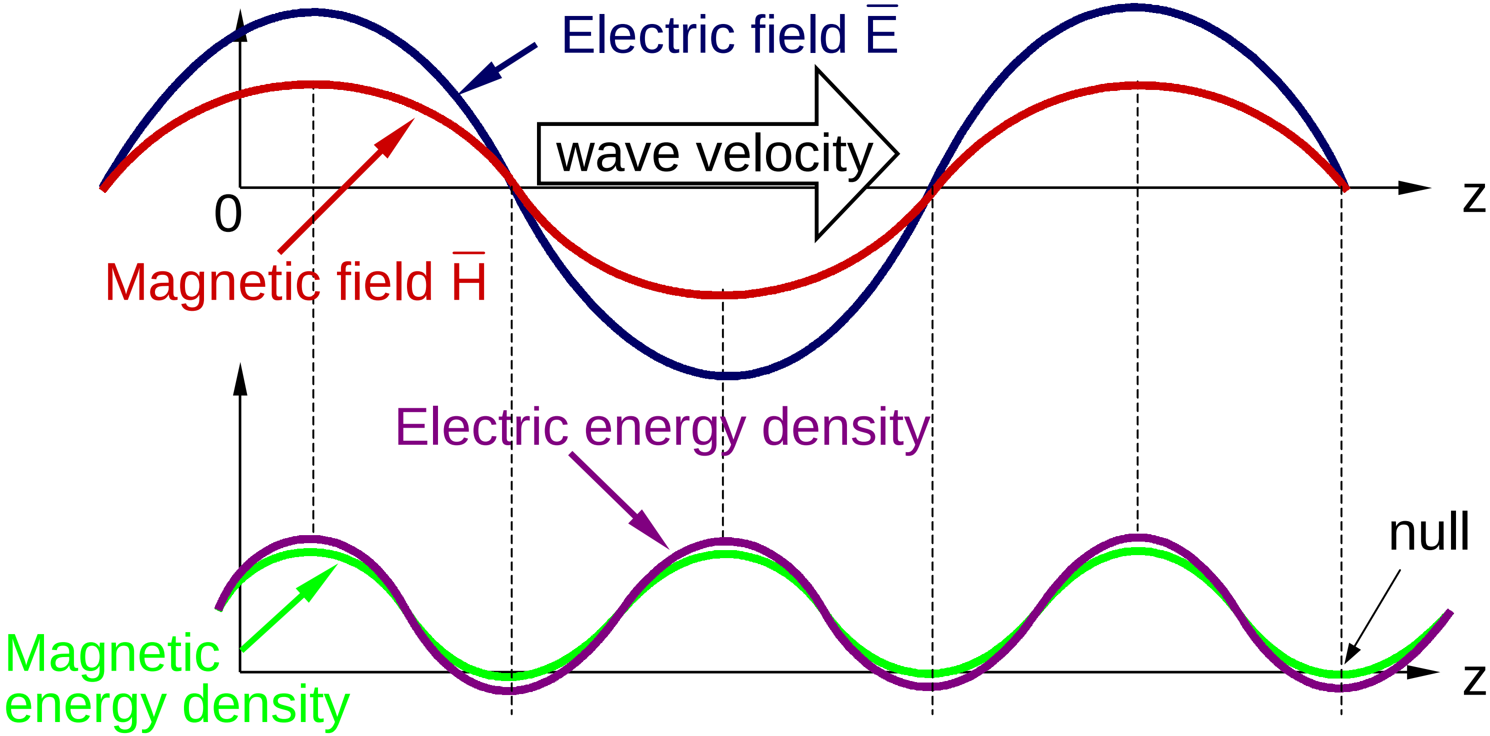
WHAT ARE ELECTROMAGNETIC WAVES?

A "wave" is a periodic disturbance propagating through a medium

EM Waves Convey Undulations in EM Fields:



Electric and Magnetic Fields are Useful Fictions:

Explain all classical electrical experiments with simple equations

4 Maxwell's equations plus the Lorentz force law

(quantum effects are separate)

WHAT ARE ELECTRIC AND MAGNETIC FIELDS?

Lorentz Force Law:

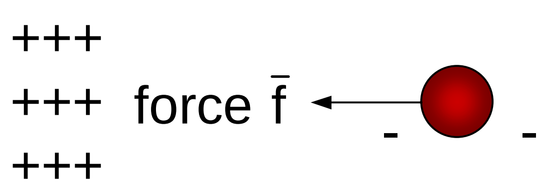
$$\overline{f} = q\left(\overline{E} + \overline{v} \times \mu_o \overline{H}\right)[\text{Newtons}]$$

$\overline{E}$	Electric field	[volts/meter; V/m]
$\overline{H}$	Magnetic field	[amperes/meter; A/m]
$\overline{f}$	Mechanical force	[newtons; N]
q	Charge on a particle	[coulombs; C]
$\overline{v}$	Particle velocity vector	[meters/second; m/sec]
μ_o	Vacuum permeability	$[1.26 \times 10^{-6}$ Henries$]$

Electric and Magnetic Fields are what Produce Force $\overline{f}$:

$\overline{f} = q\overline{E}$ when $\overline{v} = 0$, defining $\overline{E}$ via an observable

$\overline{f} = q\overline{v} \times \mu_o \overline{H}$ when $\overline{E} = 0$, defining $\overline{H}$ via an observable



MAXWELL'S EQUATIONS

Differential Form:

[1]Tesla = Webers/m^2 = 10^4 Gauss

Faraday's Law: $\nabla \times \bar{E} = -\partial \bar{B}/\partial t$

Ampere's Law: $\nabla \times \bar{H} = \bar{J} + \partial \bar{D}/\partial t$

Gauss's Laws: $\nabla \bullet \bar{D} = \rho$

$\nabla \bullet \bar{B} = 0$

$\bar{E}$	Electric field	[volts/meter; V/m]
$\bar{H}$	Magnetic field	[amperes/meter; A/m]
$\bar{B}$	Magnetic flux density	[Tesla; T][1]
$\bar{D}$	Electric displacment	[coulombs/m^2; C/m^2]
$\bar{J}$	Electric current density	[amperes/m^2; A/m^2]
ρ	Electric charge density	[coulombs/m^3; C/m^3]

Integral Form:

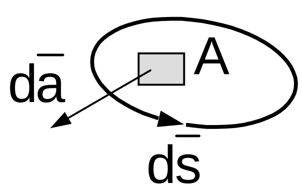
$$\int_C \bar{E} \bullet d\bar{s} = \int_A -\left(\partial \bar{B}/\partial t\right) \bullet d\bar{a}$$

$$\int_C \bar{H} \bullet d\bar{s} = \int_A \left(\bar{J} + \partial \bar{D}/\partial t\right) \bullet d\bar{a}$$

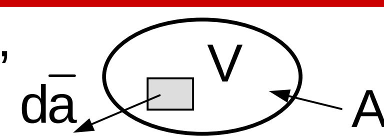
$$\int_A \bar{D} \bullet d\bar{a} = \int_V \rho \, dv$$

$$\int_A \bar{B} \bullet d\bar{a} = 0$$

Stoke's Theorem



Gauss' Law



VECTOR OPERATORS ∇, $\times$, $\bullet$

"Del" (∇) Operator: Gradient of ϕ:

$$\nabla = \hat{x}\partial/\partial x + \hat{y}\partial/\partial y + \hat{z}\partial/\partial z$$

$$\nabla\phi = \hat{x}\partial\phi/\partial x + \hat{y}\partial\phi/\partial y + \hat{z}\partial\phi/\partial z$$

"Vector Cross Product":

$$\overline{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$$

$$\overline{A}\times\overline{B} = \det\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}\left(A_yB_z - A_zB_y\right) + \hat{y}\left(A_zB_x - A_xB_z\right) + \hat{z}\left(A_xB_y - A_yB_x\right)$$

"Vector Dot Product":

$$\overline{A}\bullet\overline{B} = A_xB_x + A_yB_y + A_zB_z$$

"Divergence of $\overline{A}$":

$$\nabla\bullet\overline{A} = \partial A_x/\partial x + \partial A_y/\partial y + \partial A_z/\partial z$$

"Curl of $\overline{A}$":

$$\nabla\times\overline{A} = \det\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

"Laplacian Operator":

$$\nabla\bullet(\nabla\phi) = \nabla^2\phi = \left(\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2\right)\phi$$

MAXWELL'S EQUATIONS: VACUUM SOLUTION

Maxwell's Equations:

Faraday's Law: $\nabla \times \overline{E} = -\partial \overline{B} / \partial t$

Ampere's Law: $\nabla \times \overline{H} = \overline{J} + \partial \overline{D} / \partial t$ ($\overline{J} \rightarrow 0$)

Gauss's Laws: $\nabla \bullet \overline{D} = \rho$ ($\rho \rightarrow 0$)

$\nabla \bullet \overline{B} = 0$

$\overline{B} = \mu_o \overline{H}$

$\overline{D} = \varepsilon_o \overline{E}$

EM Wave Equation:

Eliminate $\overline{H}$: $\nabla \times (\nabla \times \overline{E}) = -\mu_o (\partial / \partial t)(\nabla \times \overline{H})$

Use identity: $\nabla \times (\nabla \times \overline{A}) = \nabla (\nabla \bullet \overline{A}) - \nabla^2 \overline{A}$

Yields: $\nabla (\nabla \bullet \overline{E}) - \nabla^2 \overline{E} = -\mu_o (\partial / \partial t)(\nabla \times \overline{H}) = -\mu_o \varepsilon_o \partial^2 \overline{E} / \partial t^2$ ($\nabla \bullet \overline{E} \rightarrow 0$)

EM Wave Equation[1] $\nabla^2 \overline{E} - \mu_o \varepsilon_o \partial^2 \overline{E} / \partial t^2 = 0$

Since:

Second derivative in space = const $\times$ second derivative in time,

Solution is any $f(\overline{r}, t)$ with identical space and time dependences

[1]Homogeneous Vector Helmholz Equation

WAVE EQUATION SOLUTION

Wave Equation has Many Solutions!

$$\nabla^2 \overline{\overline{E}} - \mu_o \varepsilon_o \partial^2 \overline{\overline{E}} \big/ \partial t^2 = 0$$

Where $\nabla^2 \phi = \left(\partial^2 / \partial x^2 + \partial^2 / \partial y^2 + \partial^2 / \partial z^2 \right) \phi$

Example:

Try: $\overline{\overline{E}} = \hat{y} E_y (z, t)$ [no x,y dependence, “UPW”]

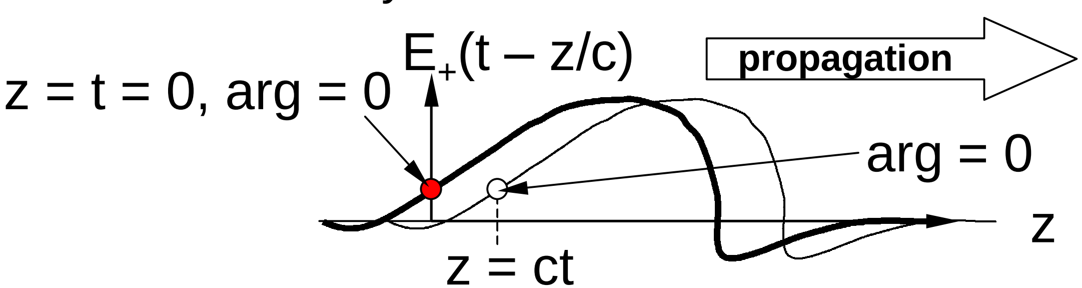
$$\left(\partial^2 / \partial z^2 \right) E_y - \mu_o \varepsilon_o \partial^2 E_y \big/ \partial t^2 = 0$$

$E_y (z, t) = E_+ (t - z/c)$, where $E_+ (.)$ is an arbitrary function

Test: $(-c)^{-2} E_+''(t - z/c) = \mu_o \varepsilon_o E_+''(t - z/c) \Rightarrow$ $c = 1 \big/ \sqrt{(\mu_o \varepsilon_o)}$

Generally: $c \cong 3 \times 10^8$ [m/s] in vacuum

$E_y (z, t) = E_+ (t - z/c) + E_- (t + z/c)$ more generally



The position • where arg = 0 moves at velocity c

UNIFORM PLANE WAVE IN Z-DIRECTION

Electric Fields (Example): $E_y(z,t) = E_+(t - z/c)$ [V/m]

More specifically, let: $E_y(z,t) = E_+ \cos \omega(t - z/c) = E_+ \cos(\omega t - kz)$,
where $k = \omega/c = \omega\sqrt{\mu_o \varepsilon_o}$

Magnetic Fields: Use Faraday's Law: $\nabla \times \overline{E} = -\partial \overline{B}/\partial t$

$$\nabla \times \overline{E} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x \to 0 & \partial/\partial y \to 0 & \partial/\partial z \\ E_x \to 0 & E_y & E_z \to 0 \end{vmatrix} = -\hat{x}\partial E_+ \cos(\omega t - kz)/\partial z$$

$$= -\hat{x} k E_+ \sin(\omega t - kz)$$

$$\overline{H}(z,t) = \hat{x}(1/\mu_o)\int k E_+ \sin(\omega t - kz)\,dt \quad [\text{A/m}]$$

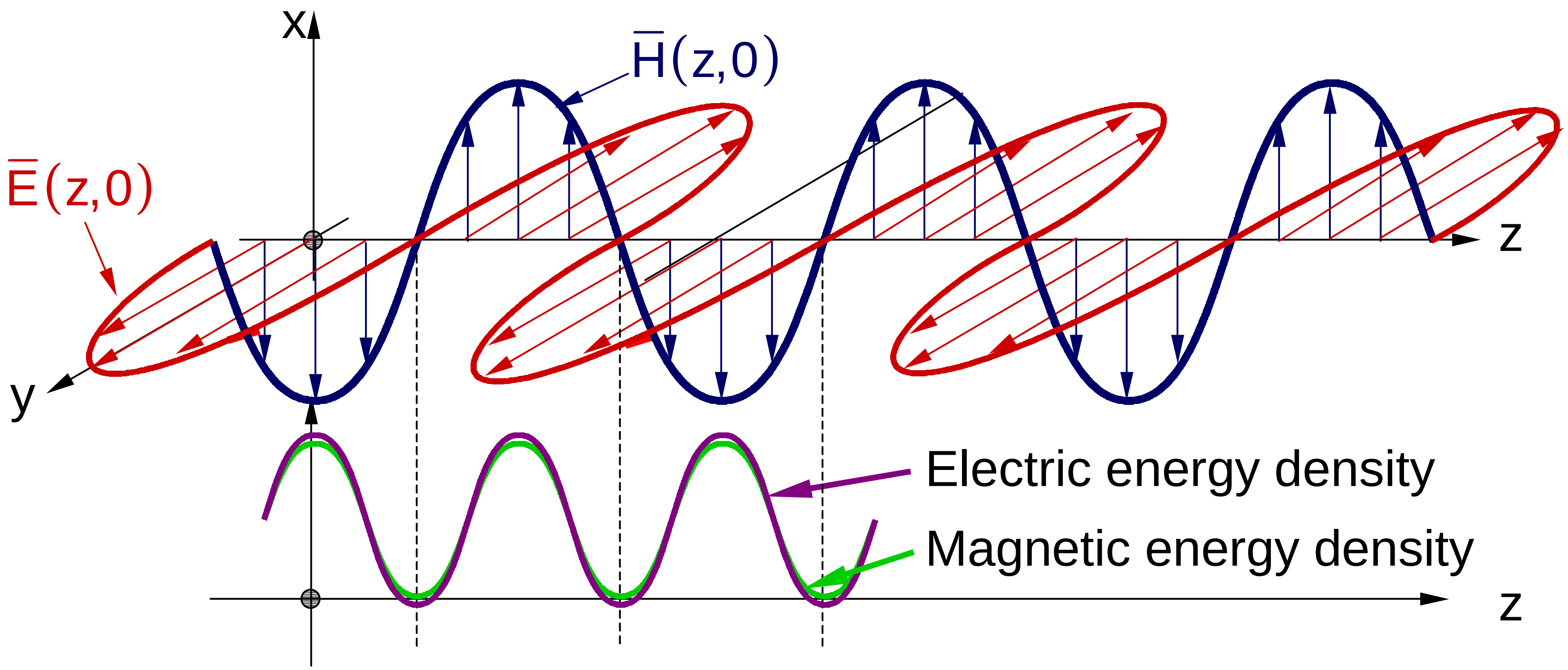
$$= -\hat{x}(kE_+/\omega\mu_o)\cos(\omega t - kz) = -\hat{x}(E_+/\eta_o)\cos(\omega t - kz)$$

$$\left[\frac{k}{\omega} = \frac{1}{c} = \sqrt{\mu_o \varepsilon_o} \;\; ; \;\; \eta_o = \sqrt{\mu_o/\varepsilon_o}\right]$$

UNIFORM PLANE WAVE EM FIELDS

EM Wave in z direction:

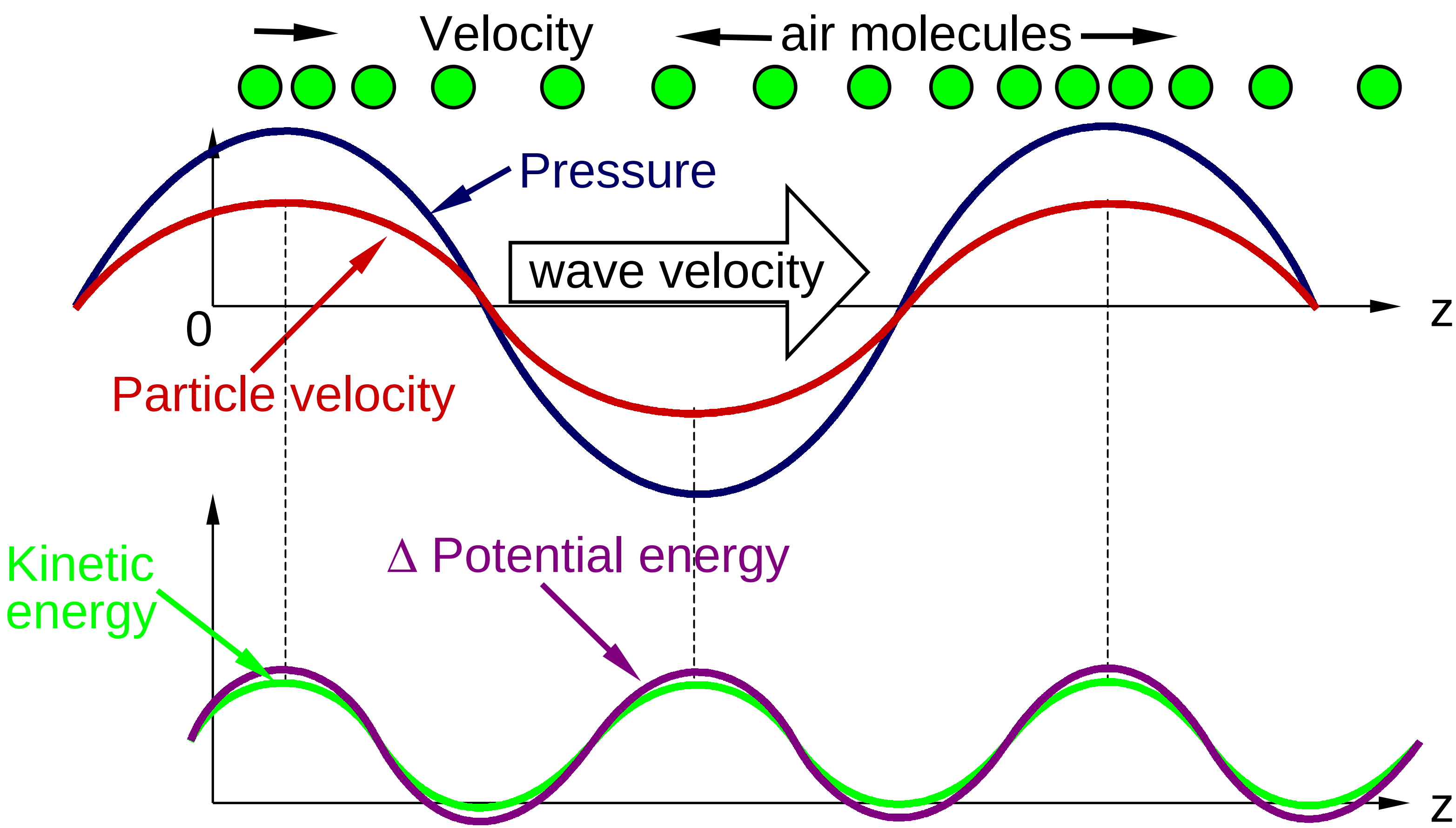
$$\overline{E}(z,t) = \hat{y}E_+(\omega t - kz)\ ,\ \ \overline{H}(z,t) = -\hat{x}(E_+/\eta_o)\cos(\omega t - kz)$$



ACOUSTIC WAVES

A "wave" is a periodic disturbance propagating through a medium

Acoustic Waves:



Generally, two types of energy are interchanged