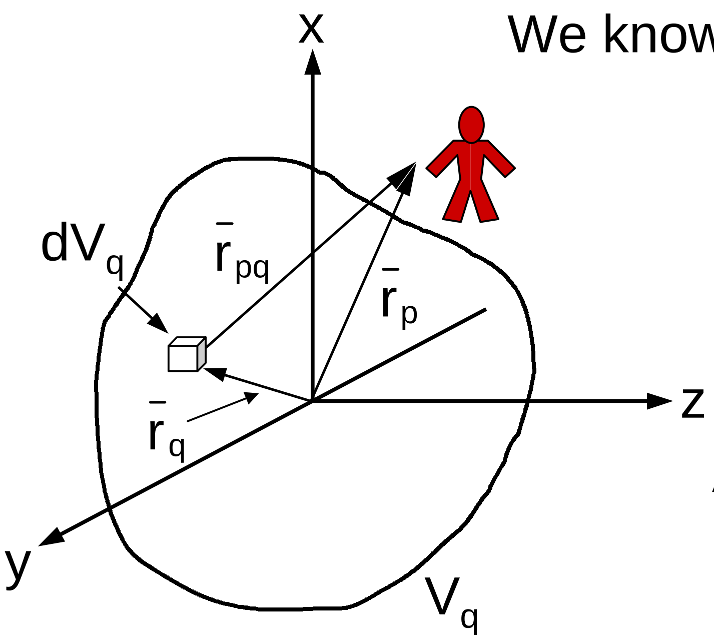
RADIATION BY CURRENTS

How are waves generated? How do we build antennas?



We know: $\nabla \bullet \bar{B} = 0$, Therefore $\bar{B} = \nabla \times \bar{A}$

$$\bar{A} = (\mu_o/4\pi) \int_{V_q} J_q \left(t - r_{pq}/c\right) r_{pq}^{-1} dv_q$$

$\nabla \times \bar{H} = \bar{J} + \partial \bar{D}/\partial t$, so $\bar{E} = +\varepsilon^{-1} \int (\nabla \times \bar{H}) dt$

Algorithm is: $\bar{J}(t, \bar{r}) \rightarrow \bar{A} \rightarrow \bar{H} \rightarrow \bar{E}$

Simplify:

Since radiation = f(ω), switch to s.s.s. (sinusoidal steady state)

Sinusoidal Steady State:

$$x(t) = R_e\{\underline{x} e^{j\omega t}\}$$
$$= R_e\{[R_e\{\underline{x}\} + jI_m\{\underline{x}\}][\cos\omega t + j\sin\omega t]\}$$
$$= R_e\{\underline{x}\}\cos\omega t - I_m\{\underline{x}\}\sin\omega t$$

EXPRESSIONS FOR RADIATION

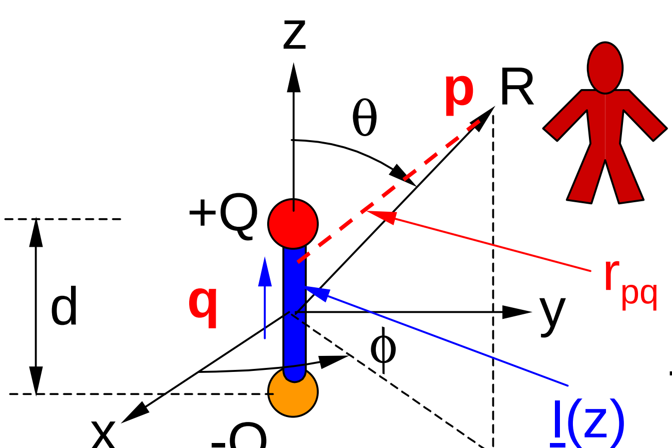
Sinusoidal Steady State Expressions:

$$\underline{\overline{B}} = \nabla \times \underline{\overline{A}}$$

$$\underline{\overline{A}} = (\mu_o/4\pi)\int_{V_q} \underline{\overline{J}}_q e^{-j\overline{k}\bullet\overline{r}_{pq}} \; r_{pq}^{-1} \; dv_q$$

$$\nabla \times \underline{\overline{H}} = \underline{\overline{J}} + j\omega\,\underline{\overline{D}}, \text{ so } \underline{\overline{E}} = \left(\nabla \times \underline{\overline{H}}\right)/j\omega\varepsilon$$

Assume short current element $\underline{I}$, $d \ll \lambda/2\pi$:



Let $\quad d \ll \lambda/2\pi,$

Then $\quad e^{-j\overline{k}\bullet\overline{r}} \cong e^{-jkR} \qquad (k = 2\pi/\lambda)$

Note: $\quad \int_{V_q} \underline{\overline{J}}\; d\,v_q = \int_{V_q} \underline{I}(z)dz = \underline{I}_o d$

Therefore:

$$\underline{\overline{A}} = (\mu_o/4\pi)\int_{V_q} \underline{\overline{J}}_q e^{-j\overline{k}\bullet\overline{r}} \; r_{pq}^{-1} \; dv_q$$

$$= (\mu_o/4\pi R)e^{-jkR}\int_{V_q} \underline{I}(z)dz$$

$$\underline{\overline{A}} = \hat{z}\mu_o \underline{I}_o d\; e^{-jkR}/4\pi R$$

EXPRESSIONS FOR RADIATION

Finding radiated $\overline{H}$:

We saw: $\overline{\underline{A}}(r) = \hat{z}\mu_o \underline{I}_o d\, e^{-jkr}/4\pi r$ (r = distance to origin)

Then: $\overline{\underline{H}} = \nabla \times \overline{\underline{A}}/\mu_o$

Where:

$$\nabla \times \overline{\underline{A}} = \left(r^2 \sin\theta\right)^{-1} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \to 0 \\ A_r & rA_\theta & r\sin\theta A_\phi \to 0 \end{vmatrix}$$

Therefore:

$$\overline{\underline{H}} = \left(\hat{\phi}/\mu_o r\right)\left[\partial\left(rA_\theta\right)/\partial r \;-\; \partial A_r/\partial\theta\right]$$

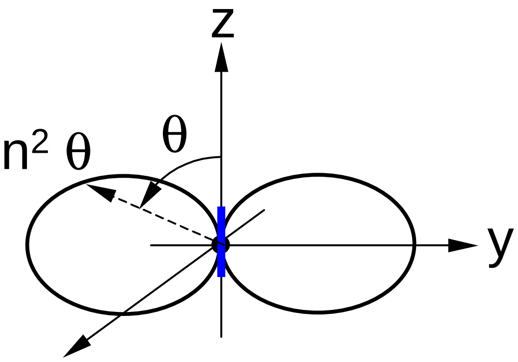
$$= \hat{\phi}\left(j\underline{I}\, d\, e^{-jkr}/2\lambda r\right)\left[1 + (jkr)^{-1}\right]\sin\theta$$

EXPRESSIONS FOR RADIATION

Finding radiated $\bar{\underline{E}}$:

Recall: $\bar{\underline{E}} = (\nabla \times \underline{H})/j\omega\varepsilon$

$P(\theta) \propto \sin^2\theta$



Therefore: $\bar{\underline{E}} = \left(j\,\underline{I}de^{-jkr}/2\lambda r\right)\eta_o$

$$\left\{\hat{\theta}\left[1+(jkr)^{-1}+(jkr)^{-2}\right]\sin\theta + \hat{r}\left[(jkr)^{-1}+(jkr)^{-2}\right]2\cos\theta\right\}$$

Radiated power:

$$\hat{r}P(\theta,\phi,r)[\mathrm{Wm}^{-2}] = 0.5R_e\{\bar{S}\} = 0.5R_e\left\{\bar{\underline{E}} \times \underline{\bar{H}}^*\right\}$$

So: $\hat{r}P(\theta,\phi,r) \cong \hat{r}\,(\eta_o/2)\left|\underline{I}d/2\lambda r\right|^2 \sin^2\theta\left[\mathrm{Wm}^{-2}\right]$ in far field, kr >>1

Where: $\underline{\bar{H}} = \hat{\phi}\left[j\underline{I}d\,e^{-jkr}/2\lambda r\right]\left[1+(jkr)^{-1}\right]\sin\theta$

$\eta_o = (\mu_o/\varepsilon_o)^{0.5}$ free-space impedance (377 ohms)

RADIATION: FAR FIELDS

Recall the Radiated Fields:

$$\overline{\underline{H}} = \hat{\phi}\left(j\underline{I}\,d\,e^{-jkr}\big/2\lambda r\right)\left[1 + (jkr)^{-1}\right]\sin\theta$$

$$\overline{\underline{E}} = \left(j\underline{I}\,d\,e^{-jkr}\big/2\lambda r\right)(\mu_o/\varepsilon_o)^{0.5}$$

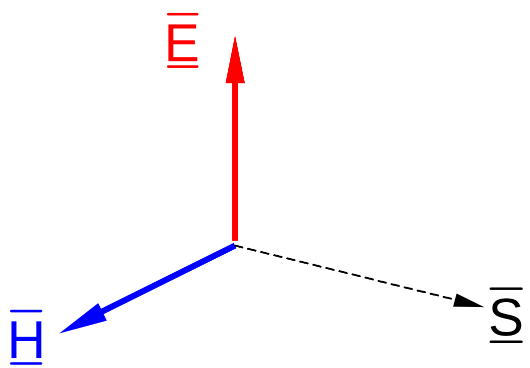
$$\left\{\hat{\theta}\left[1 + (jkr)^{-1} + (jkr)^{-2}\right]\sin\theta + \hat{r}\left[(jkr)^{-1} + (jkr)^{-2}\right]2\cos\theta\right\}$$

Far-Field Limit (kr >> 1):

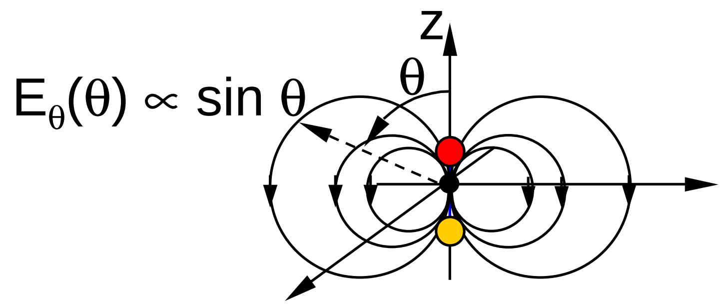
$(r >> \lambda/2\pi)$

$$\overline{\underline{E}}_{ff} \cong \hat{\theta}\sin\theta\left(j\underline{I}\,d\,e^{-jkr}\big/2\lambda r\right)(\mu_o/\varepsilon_o)^{0.5}$$

$$\overline{\underline{H}}_{ff} \cong \hat{\phi}\sin\theta\left(j\underline{I}\,d\,e^{-jkr}\big/2\lambda r\right)$$



RADIATION: NEAR FIELDS



kr << 1 ⇒ Static Dipole Fields:

$$\underline{\overline{E}} = \left(j\underline{I}d e^{-jkr} / 2\lambda r \right) (\mu_o / \varepsilon_o)^{0.5}$$

$$\left\{ \hat{\theta} \left[1 + (jkr)^{-1} + (jkr)^{-2} \right] \sin\theta + \hat{r} \left[(jkr)^{-1} + (jkr)^{-2} \right] 2\cos\theta \right\}$$

$$\underline{\overline{E}} \cong \left(j\underline{I}d e^{-jkr} / 2\lambda r \right) (\mu_o / \varepsilon_o)^{0.5} \left\{ \hat{\theta} (jkr)^{-2} \sin\theta + \hat{r} (jkr)^{-2} 2\cos\theta \right\}$$ near field

$$\underline{\overline{H}} = \hat{\phi} \left(jk\underline{I}d e^{-jkr} / 4\pi r \right) \left[1 + (jkr)^{-1} \right] \sin\theta$$

$$\underline{\overline{H}} \cong \hat{\phi} \left(\underline{I}d e^{-jkr} / 4\pi r^2 \right) \sin\theta$$ near field

Imaginary Power:

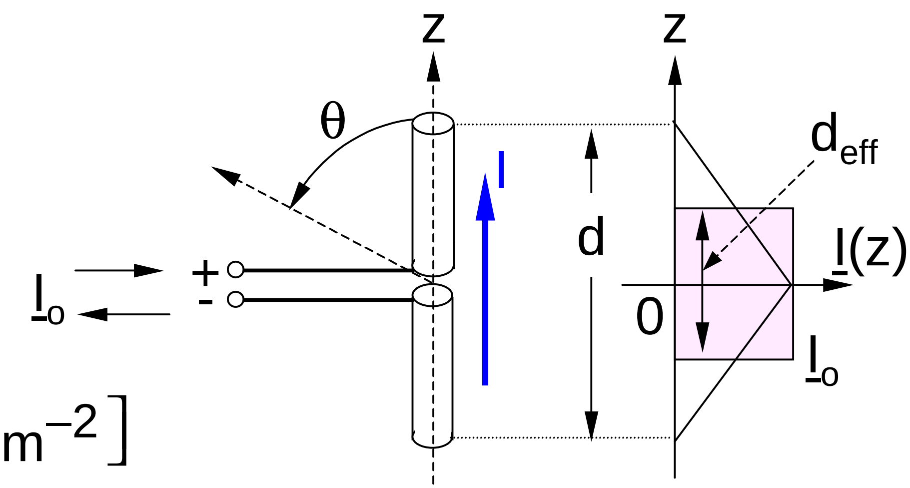
$$\underline{\overline{S}} = \underline{\overline{E}} \times \underline{\overline{H}}^* = -j\{\bullet\}$$ (average net stored energy is electric)

SHORT DIPOLE ANTENNAS

Currents on Short Dipole:

$$\int_{V_q} \overline{\underline{J}} dv_q = \int_{V_q} \overline{\underline{I}}(z)dz$$

$$\int_{V_q} \overline{\underline{J}} dv_q = \int_{V_q} \overline{\underline{I}}(z)dz = \underline{I}_o d_{eff} \cong \underline{I}_o d/2$$



Total Power Radiated:

$$P(\theta,\phi,r) = (\eta_o/2)\left|\underline{I}_o d_{eff}/2\lambda r\right|^2 \sin^2\theta \left[\mathrm{Wm}^{-2}\right]$$

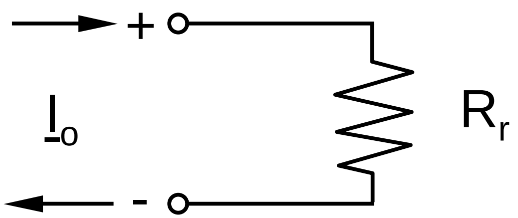
$$P_R = \int_0^{\pi}\int_0^{2\pi} P(\theta,\phi,r)(r d\phi \sin\theta) r\ d\theta = \left|k\underline{I}_o d_{eff}\right|^2 \eta_o / 12\pi\ [\mathrm{W}]$$

Radiation Resistance R_r:

$$P_R = \left|\underline{I}_o\right|^2 R_r/2\ [\mathrm{W}]$$

$$R_r = (kd)^2 \eta_o/6\pi \text{ ohms}$$

$$= (2\pi\eta_o/3)(d_{eff}/\lambda)^2$$

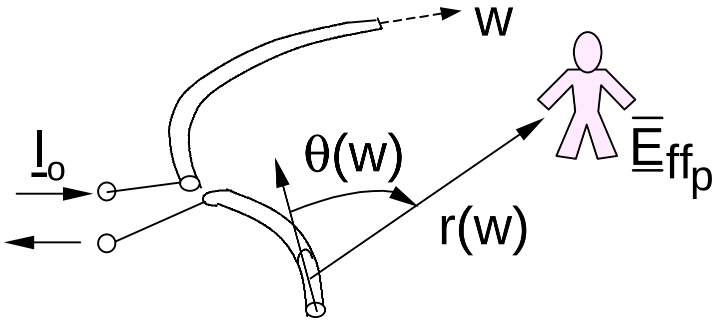


Example: 1-MHz AM signal, λ = 300 m

d_{eff} = 1 m $\Rightarrow R_r \cong$ 0.1 ohm (badly mismatched)

WIRE ANTENNAS

Sum of Short Dipole Elements:



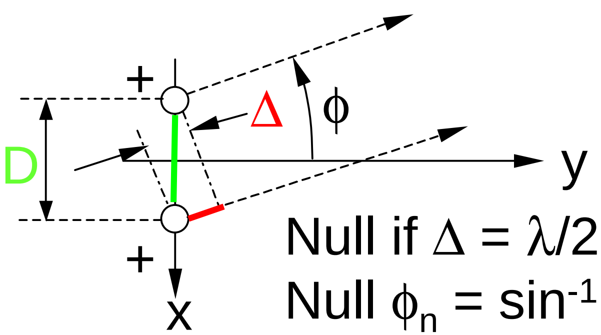
$$\underline{\overline{E}}_{ff}\, d_{eff} \cong \hat{\theta} \sin\theta \left(j \underline{I}_o d_{eff}\, e^{-jkr} / 2\lambda r \right) \eta_o d_{eff}$$

$$\overline{\underline{E}}_{ff_p} = \int_W \underline{\overline{E}}_{ff_p}(w)\, dw$$

where $\underline{I}$, r, and θ vary with w

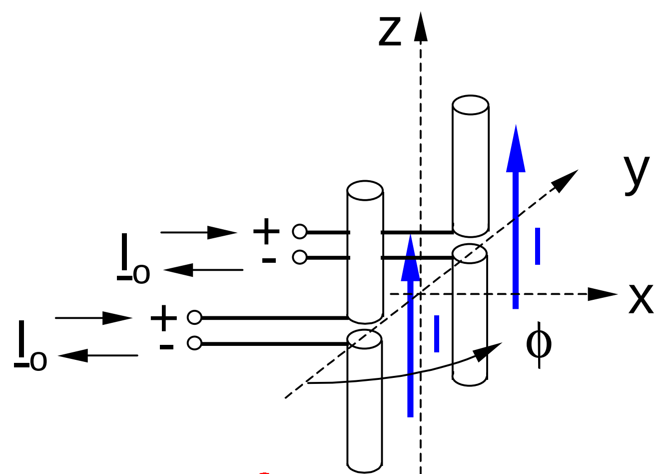
Superposition applies

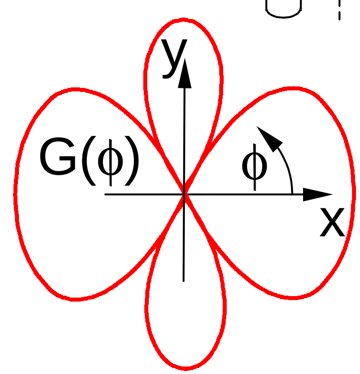
Radiation from a Pair of Dipoles:



Null if $\Delta = \lambda/2 + n\lambda$ (n = 0,1,2…)

Null $\phi_n = \sin^{-1} \Delta/D = \sin^{-1} \lambda/2D$





Example, D = λ: $\phi_n = \sin^{-1} [(\lambda/2)/\lambda] = 60°$