6.013 Recitation 25:
Gain saturation, laser oscillators and Fabry-Perot filters

Gain saturation in optical amplifiers and lasers

We have seen that the basic condition for laser operation – the threshold condition for power to build up inside the laser oscillator – is that the net gain per round trip in the laser resonator must be greater than zero. This is expressed by:

$$(1-t^2)\cdot e^{2(g-\alpha)L} \geq 1 \qquad (1)$$

where L is the distance between the two mirrors forming the oscillator, t^2 is the intensity transmission coefficient of the output mirror (at one end), α is the propagation coefficient (loss/m) of the medium between the mirrors and g is the gain coefficient (gain/m) produced by excitation of the amplifying medium. If the left hand side of this equation is greater than one, as required for build-up, the intensity between the mirrors grows with each round trip. Clearly this cannot go on forever. At some point the increasing intensity begins to saturate (deplete) the gain, g, until the left hand side of the equation becomes equal to one, and further build-up stops. At that point, steady state operation of the laser is reached. We now consider how that process occurs.

The growth in intensity of the optical wave as it propagates in the amplifying medium is given simply by:

$$\frac{dS}{dz} = gS \quad \Rightarrow \quad S = S_o e^{\int g dz} \qquad (2)$$

where the gain coefficient is $g = \sigma N$. That is, g is simply proportional to N, the density (m^{-3}) of excited atoms, and the effective cross-section $\sigma(m^2)$ of each excited atom. In these units S is the number of photons/sec/m^2.

The excited atom density N is determined by a simple equation:

$$\frac{dN}{dt} = J - \frac{N}{\tau} - \sigma NS \qquad (3)$$

where J is the rate at which atoms are excited (or, in the case of a semiconductor laser, the rate at which electrons and holes are injected) and τ is the natural lifetime of the excited atoms. Atoms are stimulated down from the excited state by the presence of the optical intensity S. In the absence of light (S = 0), or at very low levels, the steady state (d/dt = 0) density of excited atoms is simply:

$$N_o = J\cdot\tau \qquad (4)$$

In the presence of light (also in the steady state with d/dt = 0),

$$N = \frac{N_o}{1+\sigma\tau S} = \frac{N_o}{1+S/S_{sat}} \qquad (5)$$

Thus we see that the gain, $g = \sigma N$, is depressed as S increases. This is called gain saturation, and $S_{sat} = 1/\sigma\tau$ is called the saturation intensity. At $S = S_{sat}$ the gain is reduced to one half its small signal value. When the gain is depressed so that it no longer exceeds the loss in the medium, no more energy can be extracted and S experiences no further amplification. In a laser, the gain is depressed to the point where equality holds in the threshold equation.

Example: The cross-section and lifetime of the erbium atoms in an erbium-doped fiber amplifier (EDFA) are about 10^{-23} m^2 and 10^{-2} sec respectively. S_{sat} is therefore about 10^{25} photons/sec/m^2. The beam area in an optical fiber is about 10^{-8} m^2, and at a wavelength of 1.5 μm there are 7.5 x 10^{18} photons/sec/watt. (The energy per photon is hf where h = 6.626 x 10^{-34} Joule-sec is Planck's constant.) The output power at which an EDFA is saturated is therefore about 0.013 watts. Since all of the many wavelength channels in a wavelength-division-multiplexed (WDM) system pass through and saturated the same amplifiers, this power level must be sufficient to accommodate all of them. (Lasers can achieve much higher powers because the beam areas in most laser oscillators are much larger.)

Laser resonators

We now consider the properties of the oscillator that further determine the output characteristics of the laser. The basis structure of the oscillator is shown again in Figure R25.1.

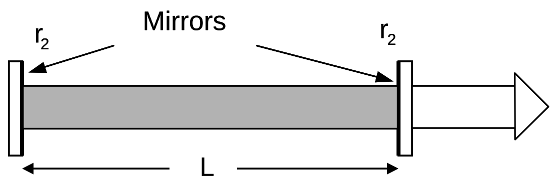

Figure R25.1: A laser oscillator

Assume first that both mirrors are perfect conductors requiring E field = zero at their front surfaces. (The magnitudes of the reflection coefficients are unity: $r_1 = r_2 = 1$) Then, the only allowed modes must satisfy the condition L = mλ/2. The frequencies of these modes are thus

$$f_m = mc/2L\,. \qquad (6)$$

Thus the allowed modes of the resonator are equally spaced in frequency by the "cavity mode spacing" c/2L. If a laser medium is placed inside the resonator, laser action may occur at one or more of these frequencies depending upon the bandwidth and other properties of the laser medium. We note that this resonance condition applies more generally for any pair of mirrors, since the existence of a resonant mode requires that the wave repeat itself after one roundtrip, i.e. that 2L be an integral number of wavelengths. In most practical oscillators, too, the mirrors are curved rather than flat, to better contain the light; but this basic resonance condition remains valid.

If we now assume that one of the mirrors has an intensity transmission t^2, the fractional loss in energy per roundtrip is also t^2. (Note: all losses per round trip of any kind may be included in t^2 for this calculation.) Since a roundtrip consists of $m\lambda$, or $m2\pi$ radians, the fractional energy lost per radian is $t^2/2m\pi$ and the Q of the mode is:

$$Q = 2m\pi / t^2 \tag{7}$$

Example: For a typical optical wavelength of 1 μm (f = $3\text{x}10^{14}$ Hz) and a resonator length of 1m, we have m = $2\text{x}10^6$. So if t^2 = 1%, the Q = $4\pi\text{x}10^8$. Because a laser oscillator is such a large number of wavelengths in length, Q can be enormous if t^2 is small. The internal losses, assumed to be zero in this calculation, must of course also be small.

Dispersion in the resonator: Now consider an optical resonator filled with a material that has dispersion, i.e. $\varepsilon = \varepsilon(\omega)$. Since the roundtrip distance 2L must always equal an integral number of wavelengths <u>in the medium</u>, and $\lambda_m = 2\pi/k_m$, we have:

$$\begin{gathered} m \cdot \frac{2\pi}{k_m} = 2L = (m+1) \cdot \frac{2\pi}{k_{m+1}} \\ \Rightarrow \; m \cdot \left(\frac{1}{k_m} - \frac{1}{k_{m+1}} \right) = \frac{1}{k_{m+1}} \quad \Rightarrow \quad m \cdot \left(\frac{1}{k_m k_{m+1}} \cdot \frac{dk}{d\omega} \cdot \Delta\omega = \frac{1}{k_{m+1}} \right) \end{gathered} \tag{8}$$

so that:

$$\Delta\omega = \frac{k_m}{m} \left(\frac{dk}{d\omega} \right)^{-1} = \frac{2\pi}{2L} \cdot v_{group} \quad \text{and} \quad \Delta f = f_{m+1} - f_m = \frac{\Delta\omega}{2\pi} = \frac{v_{group}}{2L} \tag{9}$$

We see that, in the presence of dispersion, the frequency difference between modes is proportional to the group velocity v_g, not the phase velocity c. Since any output from the laser must have a spectrum made up of some set of these (periodically spaced) cavity frequencies, the output must, in turn, be periodic in time. (The Fourier transform of a periodic function is also a periodic function.) The temporal period T = 1/Δf is just the group velocity roundtrip time of the signal traveling back and forth inside the laser. If many periodic cavity modes are excited and locked in-phase ("modelocked"), the output is a train of very short pulses with repetition rate T = 1/Δf. The duration of the pulses (about equal to the inverse of the total spectrum $\Delta\Omega = m\Delta\omega$) is given approximately by

$\tau_{pulse} \cong 1/2\pi m \Delta f$. A modelocked laser producing pulses shorter than 1 psec at a repetition rate of 100 MHz has a spectrum consisting of more than 1500 phase-locked oscillator modes.

The Fabry-Perot Etalon

Now consider an optical resonator with each mirror having amplitude reflectivity and transmission coefficients with magnitudes r and t, where by power conservation $t^2 = 1 - r^2$. Multiple reflection and transmission events occur at each mirror as illustrated in Figure R25.2

Figure R25.2: Multiple reflections in a Fabry-Perot etalon

A wave with amplitude E_o and wavevector k is incident from the left. The transmitted wave is then:

$$\begin{aligned} E_t &= E_o e^{-jkL}\left[t^2 + t^2 r^2 e^{-2jkL} + ...\right] = E_o e^{-jkL} t^2 \sum_0^\infty X^n \\ &= E_o e^{-jkL} t^2 \left(\frac{1}{1-X}\right) \end{aligned} \qquad (10)$$

where $X = r^2 e^{-2jkL}$ and $|X| < 1$. Then with $t^2 = 1 - r^2$, we have:

$$E_t = E_o e^{-jkL}\left\{\frac{1-r^2}{1-r^2 e^{-2jkL}}\right\} \text{ and } \frac{|E_t|^2}{|E_o|^2} = \frac{(1-r^2)}{1-2r^2\cos 2kL + r^4} \qquad (11)$$

A plot of this ratio of transmitted intensity to incident intensity is shown in Figure R25.3.

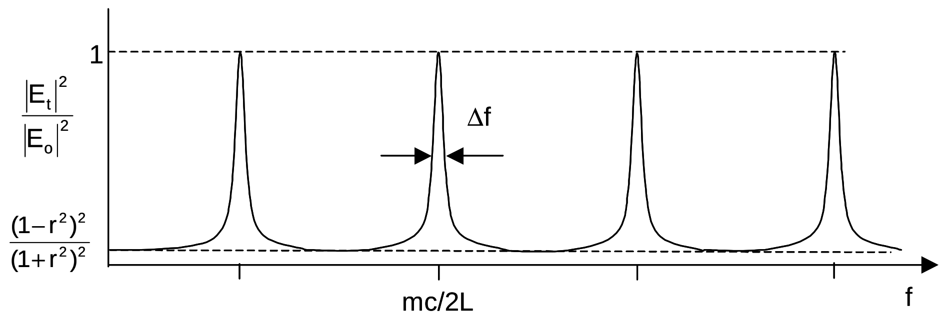

Figure R25.3: Transmission through a Fabry-Perot etalon

We note that there is 100% transmission ($\frac{|E_t|^2}{|E_o|^2}=1$) for 2kL=2mπ, i.e. at $f = m\frac{c}{2L}$.

That is: a Fabry-Perot oscillator has maximum transmission at exactly those frequencies which are resonant inside. From a physical point of view these are the only frequencies for which the intensity inside is strong enough so that the fraction emerging in the backward direction is large enough to exactly cancel the initial reflection.

If we assume that r is close to one, then the transmission peaks are narrow; and cos2kL, even at the 50% transmission point, may be approximated as $\cos 2kL \cong 1-\frac{1}{2}(2kL)^2$. The frequency width (full width at half maximum) of each transmission peak is then given by:

$$\Delta f = \frac{c}{2L}\cdot\frac{(1-r^2)}{\pi\cdot r} \quad \cong \quad \frac{f_m}{m}\frac{t^2}{\pi} \tag{12}$$

This is exactly what we would have predicted from our determination previously of the resonator $Q = \frac{m\pi}{t^2}$.

Because the widths of the transmission peaks can be very narrow, and because transmission can be 100% at the peaks, Fabry-Perot etalons are widely used as optical pass-band filters. If one only wants to transmit a single frequency band, however, it is obvious that the spacing between transmission peaks must be greater than the total width of the incident spectrum. The spacing between transmission peaks, given by c/2L, is called the "free spectral range" of the filter. Thus, a useful figure of merit for a Fabry-Perot filter is the ratio of the free spectral range to the bandwidth: $F = (c/2L)/\Delta f = \pi/t^2$. This is called the "finesse."

Example: A filter that would provide a single channel bandwidth of 50 GHz and a free spectral range equal to the full optical communication C-band (7×10^{12} Hz = 50nm, centered at 1550 nm) would need a finesse greater than 140. One disadvantage of Fabry-

Perot filters for such applications is the shape of the transmission function which (Lorentzian-like) has undesirable wings. There is currently considerable interest in designing filters with more square-like filtering functions.

It is important to note that the absolute frequency of a Fabry-Perot transmission peak is a very sensitive function of the mirror separation L. A change in L by only $\lambda/2$ (or about half a micron) moves the peak by an amount c/2L - all the way across the free spectral range. This generally requires that fixed channel filters be stabilized by either temperature control or active (piezoelectric) control, or both. For other applications this sensitivity can be a benefit. If one wants to measure an optical spectrum by scanning a transmission peak over the full free spectral range, one only has to move one of the mirrors by half a wavelength.