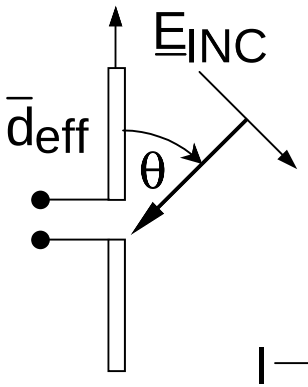
RECEIVING ANTENNAS

Recall: $A_e(\theta,\phi) = \left(\lambda^2/4\pi\right)G(\theta,\phi)$

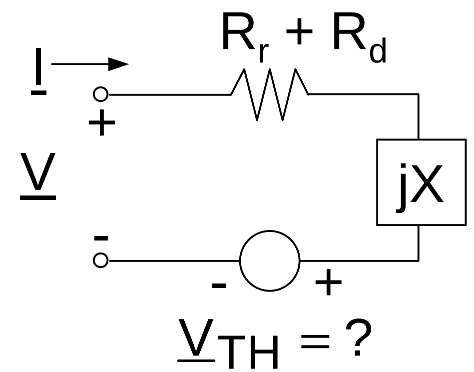
We never proved it; sometimes untrue (when?)

Proof for Short Dipole Antenna:



If $d \ll \lambda/2\pi$, quasistatics applies

We seek V_{Th} in equivalent circuit:

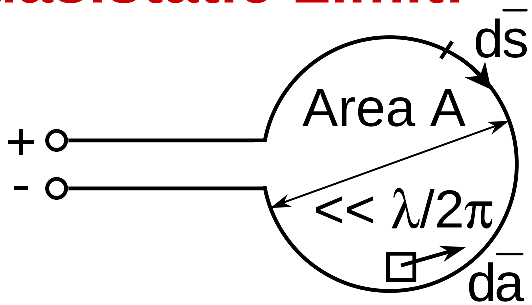


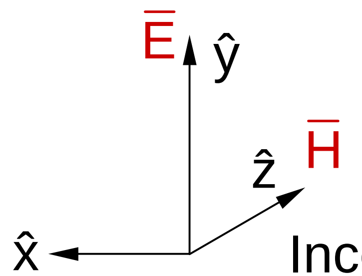
Assume Normally Incident Uniform Plane Wave:

Case 1: small loop antenna ($D \ll \lambda/2\pi$)

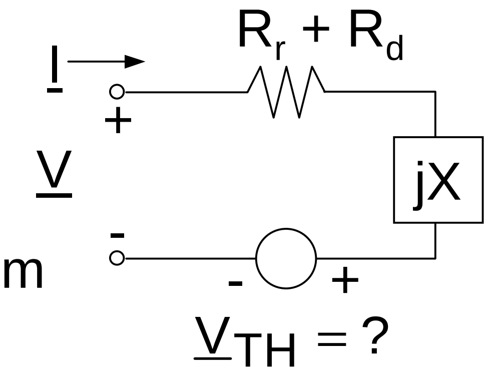
Case 2: short dipole antenna ($d \ll \lambda/2\pi$)

Quasistatic Limit:





Incoming uniform plane wave



Faraday's Law:

$$\nabla \times \overline{E} = -\partial \overline{B}/\partial t \Rightarrow -\frac{d}{dt}\int_A \overline{B} \bullet d\overline{a} = \int_c \overline{E} \bullet d\overline{s} = V_{Th}$$

Open Circuit Voltage:

$$V_{Th} = -A\mu_o \left(\partial\left[\overline{H} \bullet \hat{z}\right]/\partial t\right)$$

UPW: Power $P\left[W/m^2\right] = \eta_o \left|\overline{H}_o\right|^2/2$

Where: $\overline{H} = \hat{z}H_o \cos\omega t$ at $z = 0$, and

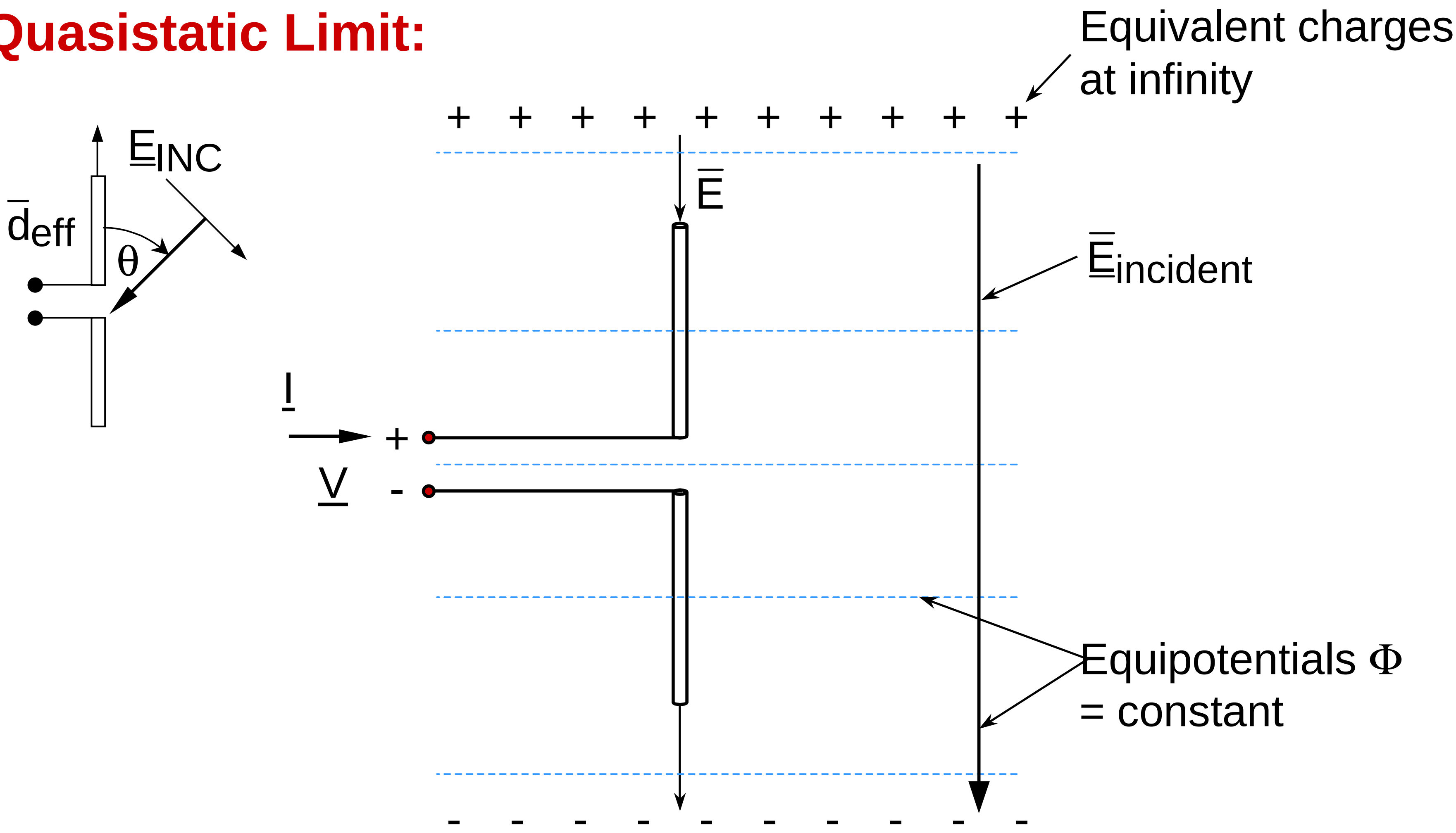
$$\left(\partial\left[\overline{H} \bullet \hat{z}\right]/\partial t\right) = -\omega H_o \sin\omega t$$

Therefore: $\left(\partial\left[\overline{H} \bullet \hat{z}\right]/\partial t\right) = -\omega\left(2P/\eta_o\right)^{0.5} \sin\omega t$ and

$$V_{Th} = A\mu_o\omega\left(2P/\eta_o\right)^{0.5} \sin\omega t$$

SHORT DIPOLE ANTENNA: OPEN CIRCUIT VOLTAGE

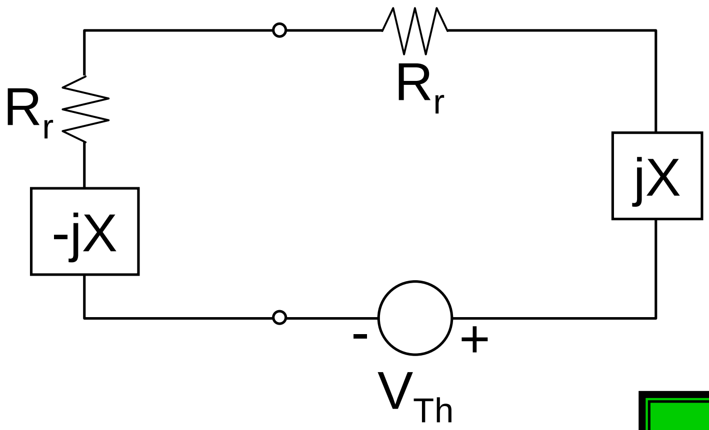
Quasistatic Limit:



$$\underline{V}_{Th} = -\overline{\underline{E}}_{INC} \bullet \overline{d}_{eff} = -\overline{\underline{E}}_{INC} d_{eff} \sin\theta$$

MAXIMUM POWER EXTRACTABLE FROM A SHORT DIPOLE ANTENNA

Antenna Equivalent Circuit plus Matched Load:



$$P_{rec.max} = \frac{1}{2}\left|\underline{V}_{Th}/2\right|^2 / R_r$$

$$= \left|\overline{\underline{E}}_{INC}\right|^2 \frac{d_{eff}^2}{8R_r} \sin^2\theta$$

where $R_r = \eta_o (kd_{eff})^2 / 6\pi$

Define $A_{eff}(\theta,\phi) \triangleq \dfrac{P_{rec}}{\left|\overline{\underline{E}}_{inc}\right|^2 / 2\eta_o}$ "Effective Area"

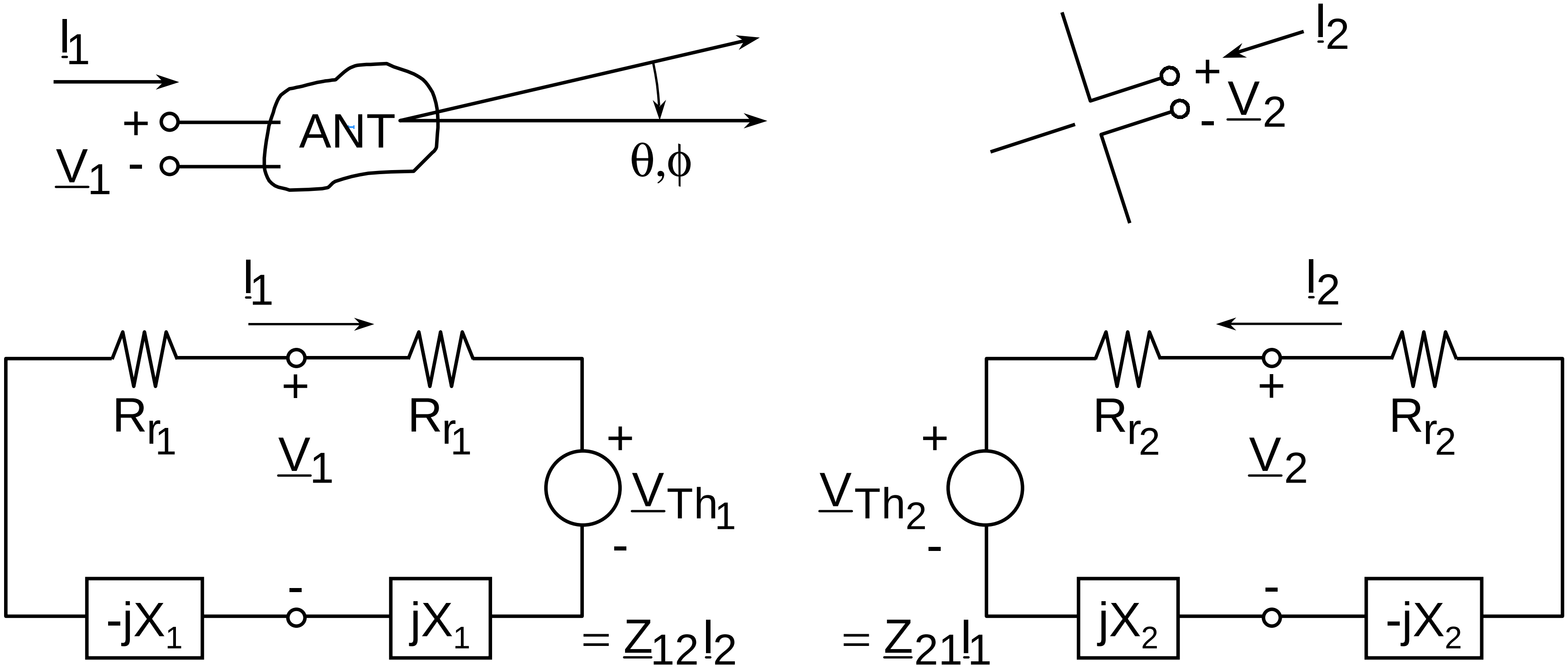
For short dipole: $A_{eff}(\theta) = \dfrac{2\eta_o P_{rec}}{\left|\overline{\underline{E}}_{inc}\right|^2} = \dfrac{\eta_o d_{eff}^2 \sin^2\theta}{4R_r} = \dfrac{\lambda^2 3}{8\pi}\sin^2\theta$

Therefore: $A_e(\theta,\phi) = \dfrac{\lambda^2}{4\pi} G(\theta,\phi)(m^2)$ [true for almost all antennas]

Recall, for short dipole: $G(\theta) = 1.5\sin^2\theta \quad [\neq f(\omega)]$

PROOF THAT A = Gλ²/4π FOR MOST ANTENNAS

Test Range = Unknown plus Short Dipole Antenna:



$$P_{rec_1} = \frac{\left|\underline{Z}_{12}\underline{I}_2\right|^2}{8R_{r_1}} = P_{t_2}\frac{G_2}{4\pi r^2}A_1$$

$$P_{rec_2} = \frac{\left|\underline{Z}_{21}\underline{I}_1\right|^2}{8R_{r_2}} = P_{t_1}\frac{G_1}{4\pi r^2}A_2$$

(Power transmitted by antenna #1)

Wm^{-2} at antenna 2

PROOF THAT A = $G\lambda^2/4\pi$ FOR MOST ANTENNAS (2)

$$P_{rec_1} = \frac{\left|\underline{Z}_{12}\underline{I}_2\right|^2}{8R_{r_1}} = P_{t_2}\frac{G_2}{4\pi r^2}A_1 \qquad P_{rec_2} = \frac{\left|\underline{Z}_{21}\underline{I}_1\right|^2}{8R_{r_2}} = P_{t_1}\frac{G_1}{4\pi r^2}A_2$$

$$\therefore \quad \frac{P_{rec_2}}{P_{rec_1}} = \frac{G_1 A_2 P_{t_1}}{G_2 A_1 P_{t_2}} \Rightarrow \frac{A_1}{G_1} = \frac{A_2}{G_2}\left[\frac{P_{t_1}}{P_{t_2}} \bullet \frac{P_{rec_1}}{P_{rec_2}}\right]$$

$$\text{But} \quad \frac{P_{rec_1}}{P_{rec_2}} = \frac{\left|\underline{Z}_{12}\underline{I}_2\right|^2}{\left|\underline{Z}_{21}\underline{I}_1\right|^2} \bullet \frac{R_{r_2}}{R_{r_1}} = \frac{\left|Z_{12}\right|^2}{\left|Z_{21}\right|^2} \bullet \frac{P_{t_2}}{P_{t_1}}$$

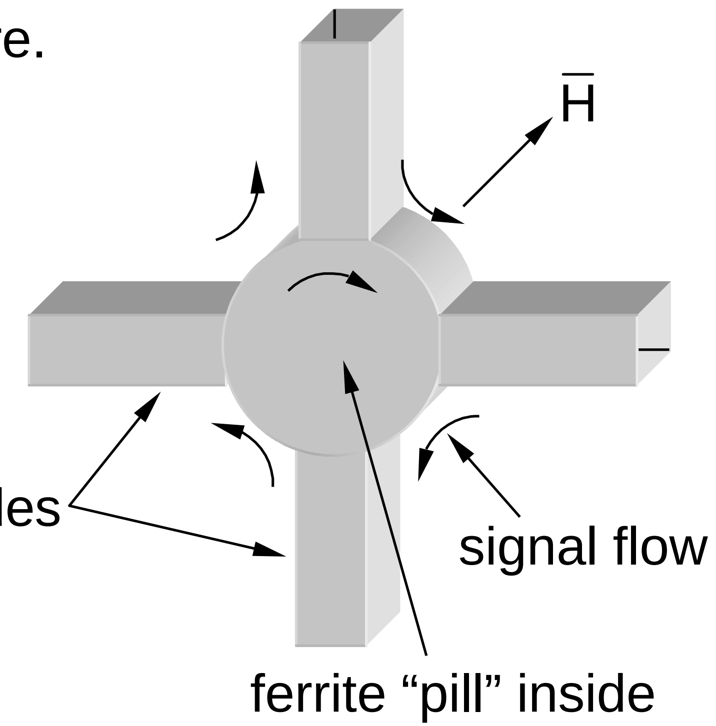
Therefore, if $\left|Z_{12}\right|^2 = \left|Z_{21}\right|^2$, then $\frac{A_1}{G_1} = \frac{A_2}{G_2} = \frac{\lambda^2}{4\pi}$ Q.E.D.

RECIPROCITY AND NON-RECIPROCAL DEVICES

Reciprocity:

$$|\underline{\underline{Z}}_{12}|^2 = |\underline{\underline{Z}}_{21}|^2 \quad \text{if} \quad \bar{\bar{\varepsilon}} = \bar{\bar{\varepsilon}}_t \; , \; \bar{\bar{\mu}} = \bar{\bar{\mu}}^t \quad \text{everywhere.}$$

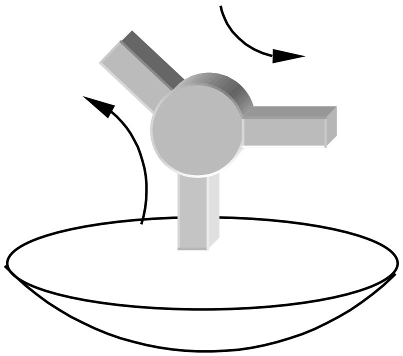
Exceptions: magnetized plasmas,
magnetized ferrites



Non-reciprocal Devices:

4-Port Circulators

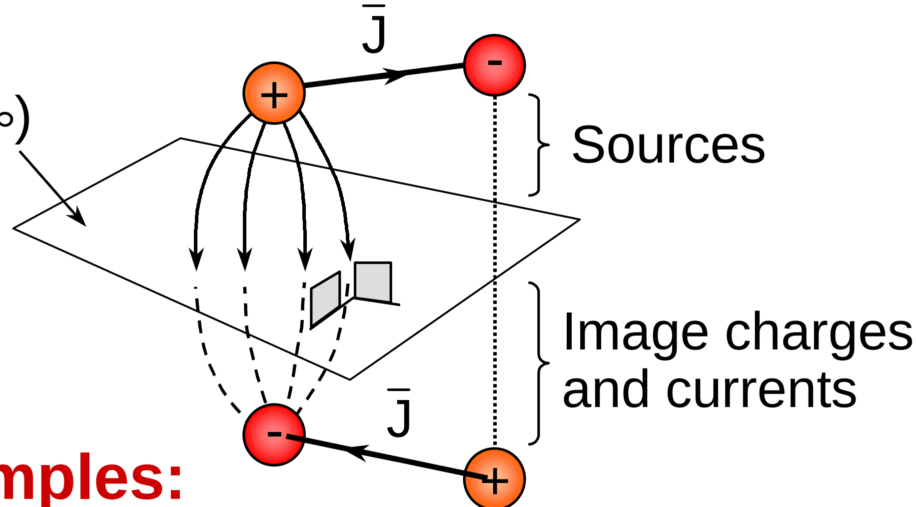
Non-reciprocal Antennas:

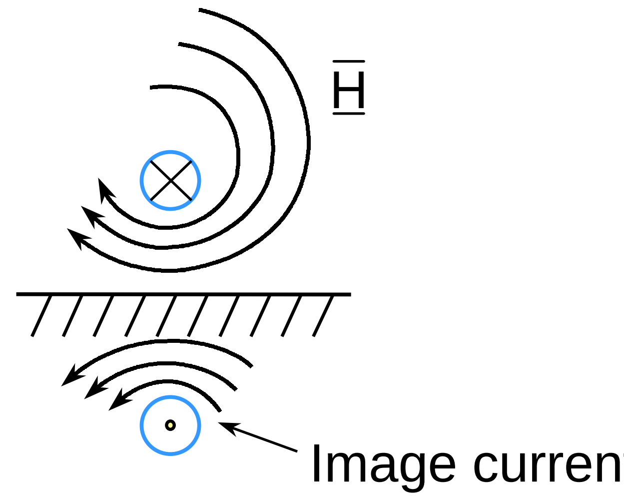


MIRROR IMAGES

Mirror Images: Infinite flat conducting surfaces are “mirrors”

Consider the charges and currents shown, and how $\overline{E}$ must everywhere be perpendicular to the mirror, and $\overline{H}$ must be parallel





Examples: