

6.431 Spring 2005 Quiz 1
Monday, March 7, 12:05-12:55 p.m.

DO NOT TURN THIS QUIZ OVER UNTIL
YOU ARE TOLD TO DO SO

- You have 50 minutes to complete the quiz.
- Write your solutions in the exam booklet. We will not consider any work not in the exam booklet.
- This quiz has three problems that are not necessarily in order of difficulty.
- You may give an answer in the form of an arithmetic expression (sums, products, ratios, factorials) of numbers that could be evaluated using a calculator. Expressions like $\binom{8}{3}$ or $\sum_{k=0}^5 (1/2)^k$ are also fine.
- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgment of your level of understanding as reflected by what you have written.
- This is a closed-book exam except for one double-sided, handwritten, 8.5 by 11 formula sheet plus a calculator.
- Be neat! If we can't read it, we can't grade it.
- At the end of the quiz, turn in your solutions along with this quiz (this piece of paper).

Write your name, your recitation instructor's name, and your TA's name on the front of the booklet. (3 points)

Problem 1: (30 points)

Consider the following game. You throw two fair coins independently. Let's refer to the coins as coin 1 and coin 2. If they have the same outcome, you win; otherwise, you lose.

- (a) (15 points) Is the outcome of coin 1 independent of whether you win or lose? Is the outcome of coin 2 independent of whether you win or lose? Explain your answer.
- (b) (15 points) Are the outcomes of the two coins independent of each other conditioned on your having won or lost? Explain your answer.

Problem 2: (50 points)

You have N urns, labeled from 1 to N , each containing $N+1$ balls. The k -th urn ($k = 1, \dots, N$) contains exactly k white balls and $N + 1 - k$ black balls. You blindly extract one ball from each urn, and mix all the extracted balls together.

- (a) (25 points) Find the mean and the variance of the number of white balls extracted. Give a simple interpretation of the value of the mean.
- (b) (10 points) What is the probability of extracting all white balls, except a black one from urn k ?
- (c) (15 points) Given that all the extracted balls except one are white, what is the probability that the black ball was originally in urn k ? You may leave the result as an algebraic sum, but try to simplify the answer as much as possible.

Formulas you *may* find useful:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}.$$

Problem 3: (17 points)

In your backpack there's a fruit left from yesterday, that's either an apple or an orange with equal probability. This morning, before coming to school, you put an apple in your backpack (without looking inside). After class you get hungry, so you reach inside and grab a fruit at random, that turns out to be an apple. What's the probability that the remaining fruit is also an apple?