QUIZ 1 ANNOUNCEMENTS

Quiz 1: (closed-book, with one *handwritten* double-sided 8.5 x 11 formula sheet and calculator permitted)

Date: Monday, March 7, 2005
Time: 12:05–12:55 p.m.

Content: All topics discussed in
Lectures 1 through 7
Reading through all of Chapter 2
Recitations 1 through 6
Tutorials 1 through 3
Problem Sets 1 through 4

Optional Quiz Review Session: There will be two identical, two-hour quiz review sessions administered by the TAs. The sessions will consist of two parts. In the first hour, a concise overview of the theory will be presented. In the second hour, a set of practice problems will be solved. The quiz review is completely optional, but it is usually a good idea to attend and reinforce your understanding of the material, and perhaps gain some insight you did not have before. Details for the quiz review sessions are:

Date: Friday, March 4, 2005
Time: 5–7 p.m. and 7–9 p.m. (identical sessions)

The problems to be solved in the quiz review are attached. We strongly recommend working through the quiz review problems before coming to the quiz review.

QUIZ 1 REVIEW PROBLEMS

1. The newest invention of the 6.041/6.431 staff is a three-sided die. On any roll of this die, the outcome x is:

$$p_X(x) = \begin{cases} \frac{1}{2}, & \text{if } x = 1 \\ \frac{1}{4}, & \text{if } x = 2 \\ \frac{1}{4}, & \text{if } x = 3 \\ 0 & \text{otherwise.} \end{cases}$$

Consider a sequence of six independent rolls of this die, and let x_i be the random variable corresponding to the ith roll.

(a) What is the probability that exactly three of the rolls have an outcome equal to 3?

(b) What is the probability that the first roll is 1, given that exactly two of the six rolls had an outcome of 1?

(c) We are told that exactly three of the rolls resulted in 1 and exactly three resulted in 2. Given this information, what is the probability that the sequence 121212 resulted?

(d) Conditioned on the event that at least one roll resulted in 3, find the conditional PMF of the number of 3's.

2. The president of a company discovers that one of her two vice presidents, A and B is embezzling money from the company. In order to determine who is guilty, she decides to hire a private detective to investigate. If she chooses to investigate VP A she will have to pay D_A to the detective, and if A turns out to be guilty, the president will have to pay R_A to replace A. Similarly, investigating B has costs D_B and R_B. Furthermore, if the detective decides that one of the VP's is innocent, the president will have to pay the detective to investigate the other VP. If the a priori probability that A is guilty is p, and that B is guilty is $1 - p$, find the conditions on p, D_A, D_B, R_A, R_B for which investigating A first would minimize the expected cost of the procedure.

3. Professor May B. Right often has her science facts wrong, and answers each of her students' questions incorrectly with probability 1/4, independently of other questions. In each lecture May is asked either 1 or 2 questions with equal probability.

   (a) What is the probability that May gives wrong answers to all the questions she gets in a given lecture?

   (b) Given that May gave wrong answers to all the questions she got in a given lecture, what is the probability that she got two questions?

   (c) Let X and Y be the number of questions May gets and the number of questions she answers correctly in a lecture, respectively. What is the mean and variance of X and the mean and the variance of Y?

   (d) Give a neatly labeled sketch of the joint PMF $p_{X,Y}(x, y)$.

   (e) Let $Z = X + 2Y$. What is the expectation and variance of Z?

   For the remaining parts of this problem, assume that May has 20 lectures each semester and each lecture is independent of any other lecture.

   (f) The university where May works has a peculiar compensation plan. Each lecture May gets paid a base salary of \$1,000 plus \$40 for each question she answers and an additional \$80 for each of these she answers correctly. In terms of random variable Z, she gets paid $\$1000 + \$40Z$ per lecture. What is the expected value and variance of her *semesterly* salary?

   (g) Determined to improve her reputation, May decides to teach an additional 20-lecture class in her specialty (math), where she answers questions incorrectly with probability 1/10 rather than 1/4. What is the expected number of questions that she will answer wrong in a randomly chosen lecture (math or science).