

Recitation 7G

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1. (Problem 3.12, page 195 in the text.) Show that the normal PDF satisfies the normalization property.

Hint: The integral $\int_{-\infty}^{+\infty} e^{-x^2/2} dx$ is equal to the square root of

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-x^2/2} e^{-y^2/2} dx dy,$$

and the latter integral can be evaluated by transforming to polar coordinates.

2. Suppose the waiting time until the next bus at a particular bus stop is exponentially distributed, with parameter $\lambda = \frac{1}{15}$. Suppose that a bus pulls out just as you arrive at the stop. Find the probability that:

(a) You wait more than 15 minutes for a bus.

(b) You wait between 15 and 30 minutes for a bus.

3. Computer chips can be expected to fail after operating for a random amount of time. Suppose, in particular, that

$$\mathbf{P}(\text{chip still works at time } t) = e^{-\alpha t}, \quad t \geq 0. \quad (1)$$

Consider now that we have a manufacturing process that produces a mix of “good” and “bad” chips. The lifetime of good chips satisfies Eq. (1). The lifetime of bad chips satisfies the same relation except that α is replaced by 1000α . Assume that the fraction of good chips is p and the fraction of bad chips $1 - p$.

(a) Find the probability that a randomly selected chip is still functioning after τ time units of operation.

(b) In order to weed out bad chips, every chip is tested for τ time units before leaving the factory, and only chips that do not fail during the testing period are shipped to customers. Give a formula for the probability that a customer receives a bad chip (as a function of the constants α , p , and τ).

4. A signal $s = 2$ is transmitted from a satellite but is corrupted by noise, so that the received signal is $X = s + W$. When the weather is good, which happens with probability $2/3$, W is normal with zero mean and variance 1. When the weather is bad, W is normal with zero mean and variance 9. In the absence of any weather information, find the PDF of X and calculate the probability that X is between 1 and 3. (Express the probability using the standard normal CDF Φ .)