**Tutorial 5 Answers**
**Week of March 7, 2005**

1. Let $G$ denote the event that Dino has a good day and let $B$ denote the event that Dino has a bad day. We are given that $\mathbf{P}(G) = \mathbf{P}(B) = 0.5$. Let $T$ be the time it takes Dino to cook a souffle, so that $f_{T|G}$ is uniform between 1/2 and 1, and $f_{T|B}$ is uniform between 1/2 and 3/2. We need to find $\mathbf{P}(B \mid T \leq 3/4)$.

   Using Bayes's rule, we have

$$\mathbf{P}(B \mid T \leq 3/4) = \frac{\mathbf{P}(T \leq 3/4 \mid B)\mathbf{P}(B)}{\mathbf{P}(T \leq 3/4)} = \frac{\mathbf{P}(T \leq 3/4 \mid B)\mathbf{P}(B)}{\mathbf{P}(T \leq 3/4 \mid B)\mathbf{P}(B) + \mathbf{P}(T \leq 3/4 \mid G)\mathbf{P}(G)}$$

   Evaluating this expression, we find that

$$\mathbf{P}(B \mid T \leq 3/4) = \frac{(\frac{1}{4})(\frac{1}{2})}{(\frac{1}{4})(\frac{1}{2}) + (\frac{1}{2})(\frac{1}{2})} = \frac{1}{3}.$$

2. a) $\gamma$ is determined because the density function must integrate to 1. Since $(X, Y)$ uniformly distributed in $R$, we have:

$$\int\int \gamma dx dy = 1$$

   where the integral is over the area of $R$. Therefore $\frac{1}{\gamma}$ =Area $R$.
   b) Showing independence is equivalent to showing that:

$$f_{XY}(x, y) = f_X(x) \cdot f_Y(y)$$

   But this is clear, since:

$$f_{XY}(x, y) = f_Y(x) = f_Y(y) = 1$$

   c) Consider a region that consists of the upper half plane, and the point $(1, -1)$. If we are told that $Y < 0$, $X$ is determined, and hence $X, Y$ cannot be independent in this region.
   d) To find the probability that $(X, Y)$ lie in the circle $C$ inscribed in the $R$ in part (b) we could integrate, or observe that the integral will in fact come out to the area of the circle, and hence the desired probability will be the ratio of the are of the circle to the area of the square:

$$\mathbf{P}((X, Y) \in C) = \frac{.25\pi}{1}.$$