Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2005)

Recitation 14
April 5, 2005

1. The shuttle bus is scheduled to arrive at 77 Mass. Ave. at 1 p.m., at 2 p.m., and at 3 p.m. However, at each hour, it only actually arrives with probability p. Whether or not the bus arrives at any particular hour is independent of what happens at any other hour.

   If the bus does arrive, then the driver flips a biased coin, with probability of a head equal to q, to decide whether to go to Kendall Square (if the flip is a head) or Harvard Square (if the flip is a tail).
   Consider the following discrete random variables for $i = 1, 2, 3$:

$$X_i = \begin{cases} 1 & \text{if a bus heads to Kendall Square at time } i, \\ 0 & \text{otherwise.} \end{cases}$$

$$Y_i = \begin{cases} 1 & \text{if a bus heads to Harvard Square at time } i, \\ 0 & \text{otherwise.} \end{cases}$$

   Also define random variables for the total arrivals to Kendall Square and total arrivals to Harvard Square:

$$X = \sum_{i=1}^{3} X_i \qquad\qquad Y = \sum_{i=1}^{3} Y_i$$

   (a) Note that X_i ($i = 1, 2, 3$) is a Bernoulli random variable, i.e., at time i, either a bus heads to Kendall or not. Compute the PMF of X_1 and the PMF of X (recall that X is the total number of buses heading to Kendall).

   (b) Compute the conditional PMF of X_1 given Y_1, and then the conditional PMF of X_1 given Y_2.

   (c) Is X_i independent of Y_i, for $i = 1, 2, 3$? Is X_i independent of Y_j, for $i, j = 1, 2, 3$ and $i \neq j$?

   (d) Let event A be defined as

$$A = \{Y_1 = 0,\, Y_2 = 1,\, Y_3 = 0\}.$$

   Compute the PMF of X_1 conditioned on the event A.

   (e) Compute the PMF of X_1 conditioned on the event $\{Y = 2\}$. (Hint: Think about your answer to part (d)).

2. Problem 5.3, on page 301 in the book (modified).
   A computer system carries out tasks submitted by two users. Time is divided into slots. A slot can be idle, with probability $p_I = 1/6$, and busy with probability $p_B = 5/6$. During a busy slot, there is probability $p_{1|B} = 2/5$ (respectively, $p_{2|B} = 3/5$) that a task from user 1 (respectively, 2) is executed. We assume that events related to different slots are independent.

   (a) Find the probability that a task from user 1 is executed for the first time during the 4th slot.

(b) Given that exactly 5 out of the first 10 slots were idle, find the probability that the 6th idle slot is slot 12.

(c) Find the expected number of busy slots up to and including the 5th task from user 1.

(d) Find the PMF of the number of tasks from user 2 until the time of the 5th task from user 1.

3. For each night, the probability of a robbery attempt at the local warehouse is $\frac{1}{5}$. A robbery attempt is successful with probability $\frac{3}{4}$, independent of the night. After any particular SUCCESSFUL robbery, the robber celebrates by taking off either the next 2 or 4 nights (with equal probability), during which time there will be no robbery attempts. After that, the robber returns to his original routine.

   (a) Let D be the number of days until (and including) the second successful robbery, including the days of celebration after the first robbery. Find the PMF of D, or its transform (whichever you find more convenient).

   During a successful robbery, the robber steals a random number of candy bars, which is 1, 2, or 3, with equal probabilities. This number is independent for each successful robbery and independent of everything else (no candy bars are stolen in unsuccessful robberies).

   (b) Let T be the number of candy bars collected in ten robbery attempts (whether successful or not). Find the PMF of T, or its transform, whichever is easier. Find the expectation and the variance of T.