Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Spring 2005)

Problem Set 2G

Due: February 16, 2005

G0. Omit Question 1 and Question 3 in Problem Set 2.

G1. Consider a community of m families and suppose, for $i = 1, \ldots, k$, that n_i of them have i children; note that $\sum_{i=1}^{k} n_i = m$. Consider the following two methods for choosing a child:

(i) Choose one of the families at random (uniformly) and then randomly (uniformly) choose a child from that family.

(ii) Choose one of the $\sum_{i=1}^{k} i n_i$ children at random.

Show that method **(i)** is more likely to result in the choice of a first-born child.

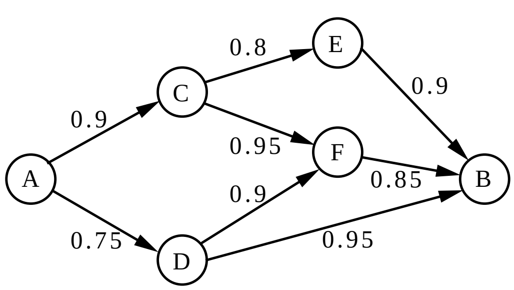
Hint: You may need to show that

$$\left(\sum_{i=1}^{k} i n_i\right)\left(\sum_{j=1}^{k} \frac{n_j}{j}\right) \geq \left(\sum_{i=1}^{k} n_i\right)\left(\sum_{j=1}^{k} n_j\right)$$

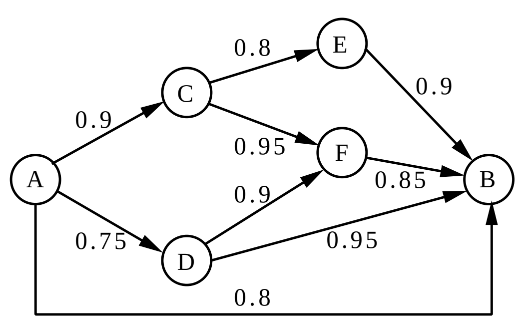
G2. A reliability problem involving a network of telephone lines need not always be decomposable into a sequence of series and parallel connections. However, the total probability theorem may still be useful in calculating the probability of successful connection between nodes. The following are some examples.

(a) In an effort to improve the reliability of connections between nodes A and B in the system of Example 1.16 in the notes, suppose a new line is added that connects nodes D and F. Assume this new line is unblocked independently of other connections with probability 0.9 (see figure (a) below, where the number next to each arc (i, j) indicates the probability that the connection from i to j is *unblocked*). What is the probability that there is at least one unblocked path connecting nodes A and B?

(b) Suppose that, in addition to the line added in part (a), another line is added that connects nodes A and B directly. This cheaper line is unblocked, also independently of other connections, with a probability of 0.8 (see figure (b) below). Now what is the probability that there is at least one unblocked path connecting nodes A and B?



(a)



(b)

G3. We have two jars each containing initially n balls. We perform four successive ball exchanges. In each exchange, we simultaneously pick at random a ball from each jar and move it to the other jar. What is the probability that at the end of the four exchanges all the balls will be in the jar where they started?