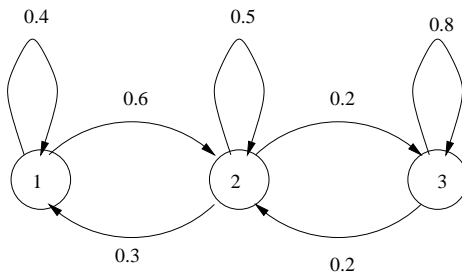


Recitation 24
May 12, 2005

1. Consider a Markov chain $\{X_n; n = 0, 1, \dots\}$, specified by the following transition diagram.



- (a) Assuming that the chain starts with $X_0 = 1$, find the probability that $X_2 = 2$.
 - (b) Find the steady-state probabilities π_1, π_2, π_3 of the different states.
 - (c) Let $Y_n = X_n - X_{n-1}$. (So, $Y_n = 1$ indicates that the n th transition was an “upwards” transition, and $Y_n = 0$ indicates a self-transition.) Find a reasonable approximation to $\mathbf{P}(Y_n = 1)$, when n is very large.
 - (d) Is the sequence Y_n a Markov chain? Explain very briefly.
 - (e) The process starts at state 1. After the first transition, you receive a reward R which is uniformly distributed between 0 and $2i$, where $i \in \{1, 2\}$ is the next state. Find the mean and variance of R .
 - (f) Does the sequence X_n converge in probability? If yes, to what?
 - (g) Does the sequence Z_n defined by $Z_n = \max\{X_1, \dots, X_n\}$ converge in probability? If yes, to what?
 - (h) Each time the state is equal to 1, we view this as a “trial”. A trial is declared to be a “success” if the next state is state 2. Determine (approximately) how large k should be in order to guarantee the following: if we observe k trials, the event {the fraction of successes is between 0.59 and 0.61} has probability at least 0.96.
2. Wombats and dingos arrive in a Poisson manner to a particular water hole in the Australian Outback. The arrival rates of wombats and dingos are 2 and 4 per hour, respectively. Each animal will stay and drink until the next animal arrives to take over. No other animals visit the water hole.
- Crocodile Dundee arrives at the water hole at a random time.
- (a) How long, on average, will Crocodile Dundee have to wait to see a dingo?
 - (b) Consider the first dingo that Crocodile Dundee sees. How long, on average, does this dingo spend at the water hole?
- Crocodile Dundee leaves the water hole immediately after the 900th animal he sees departing the water hole.

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- (c) What does Chebyshev's inequality tell you about the probability that Crocodile Dundee stays at the water hole for between 140 and 160 hours?
- (d) Using the Central Limit Theorem, what is the probability that Crocodile Dundee stays at the water hole for between 140 and 160 hours?