

**Recitation 19 - Solutions**  
**April 26, 2005**  
**Markov Processes (Section 6.3)**

1. (a) (i) True (ii) True (iii) False  
 The leftmost four states are transient and the rightmost three states are an aperiodic recurrent class.
  - (b) (i) True (ii) False (iii) False  
 There are two recurrent classes, both aperiodic. The probability you end up in either depends on the initial state.
  - (c) (i) False (ii) False (iii) False  
 The rightmost three states form a periodic recurrent class, and thus the occupancy probabilities in that class do not converge. The occupancy probabilities in the leftmost four states converge to zero.
  - (d) (i) True (ii) True (iii) True  
 The chain consists of one recurrent class and no transient states. One way to see that the recurrent class is aperiodic is to note that from the center state one can reach each state of the chain in exactly four steps.
2. Let us define the state space having 6 states, labeled 0,1,2,3,4 and 5, referring to the number of consecutive packet errors at a certain time. State 0 then corresponds to the initial state, or the state after a timeout, and state 5 corresponds to being in timeout. The transition probabilities are marked on Figure 1. As seen from the figure, this is a Markov chain with a single recurrent class, and there are no transient states. Then, the steady-state probabilities  $\pi_i$  will exist and be positive for  $i = 0, 1, \dots, 6$ . Further, they satisfy the following balance equations:

$$\begin{aligned}
 p\pi_0 &= \pi_1 \\
 p\pi_1 &= \pi_2 \\
 p\pi_2 &= \pi_3 \\
 p\pi_3 &= \pi_4 \\
 p\pi_4 + (1-q)\pi_5 &= \pi_5
 \end{aligned}$$

Using these, and  $\sum_i \pi_i = 1$ , we obtain  $\pi_0 = \frac{1}{1+p+p^2+p^3+p^4+p^5/q}$ , and the rest can be found from the equations above.

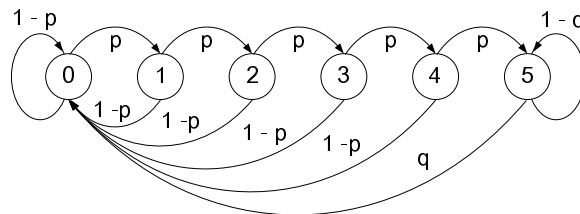


Figure 1: The Markov chain

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3. This is Problem 6.11 from the textbook, please see the online solution. The solutions for (a), (b), (c), (d), (e) are labeled as (a), (c), (d), (f), and (g) in the solution, respectively.