

Tutorial 8
Week of April 4, 2005

1. You are visiting the rainforest, but unfortunately your insect repellent has run out. As a result, at each second, a mosquito lands on your neck with probability 0.5. If one lands, with probability 0.2 it bites you, and with probability 0.8 it never bothers you, independently of other mosquitoes. What is the expected time between successive bites? What is the variance of the time between successive bites?
2. Beginning at time $t = 0$ we begin using bulbs, one at a time, to illuminate a room. Bulbs are replaced immediately upon failure. Each new bulb is selected independently by an equally likely choice between a Type-A bulb and a Type-B bulb.

The lifetime, X , of any particular bulb of a particular type is an independent random variable with the following PDF:

$$\begin{aligned} \text{For Type-A Bulbs: } f_X(x) &= \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases} \\ \text{For Type-B Bulbs: } f_X(x) &= \begin{cases} 3e^{-3x} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases} \end{aligned}$$

Parts I through III can be addressed independently.

PART I:

- (a) Find the probability, $\mathbf{P}(E)$, that the 12th bulb failure is also the 4th Type-A bulb failure.
- (b) Up to and including the 12th bulb failure, what is the probability, $\mathbf{P}(F)$, that a total of exactly 4 Type-A bulbs have failed?
- (c) Determine either the PDF or the transform for U , the time until the 12th bulb failure.

PART II:

- (d) Determine $\mathbf{P}(G)$, the probability that the *total* period of illumination provided by the first *two* Type-B bulbs is longer than that provided by the *first* Type-A bulb.

PART III:

- (e) Suppose the process terminates as soon as a total of exactly 12 bulb failures have occurred. Determine the expected value and variance for V , the total period of illumination provided by Type-B bulbs while the process is in operation.