Final Exam Announcements

Exam: (closed-book; three *handwritten* 2-sided 8.5x11 formula sheets and calculator permitted; if needed, the **Standard Normal Table** on p. 155 of the text will be provided to you)

Date/Time: Tuesday, May 17, 2005 1:30–4:30 p.m.*

Content: All topics discussed in
Lectures 1 through 23
Textbook chapters 1 through 7
Recitations 1 through 24
Tutorials 1 through 11
Problem Sets 1 through 10

*Unless you have already done so, notify the head TA if you have another exam scheduled at this date/time.

Practice Exams: Past final exams are available in the exams section for practice.

Optional Review Session: The TAs will provide a two-hour review session consisting of two parts. In the first hour, a concise overview of the theory will be presented. In the second hour, a set of practice problems will be solved. Details for the review session are as follows.

Date/Time: Friday, May 13, 2005 4–6 p.m.

The problems to be solved in the review session are attached. We strongly recommend working through the problems before coming to the review.

Review Problems

1. A 6.041 graduate opens a new casino in Las Vegas and decides to make the games more challenging from a probabilistic point of view. In a new version of roulette, each contestant spins the following kind of roulette wheel. The wheel has radius r and its perimeter is divided into 20 intervals, alternating red and black. The red intervals (along the perimeter) are twice the width of the black intervals (also along the perimeter). The red intervals all have the same length and the black intervals all have the same length. After the wheel is spun, the center of the ball is equally likely to settle in any position on the edge of the wheel; in other words, the angle of the final ball position (marked at the ball's center) along the wheel's perimeter is distributed uniformly between 0 and 2π radians.

   (a) What is the probability that the center of the ball settles in a red interval?

   (b) Let B denote the event that the center of the ball settles in a black interval. Find the conditional PDF $f_{Z|B}(z)$, where Z is the distance, along the perimeter of the roulette wheel, between the center of the ball and the edge of the interval immediately clockwise from the center of the ball?

(c) What is the unconditional PDF $f_Z(z)$?

Another attraction of the casino is the Gaussian slot machine. In this game, the machine produces independent identically distributed (IID) numbers $X_1, X_2, ...$ that have normal distribution $\mathcal{N}(0, \sigma^2)$. For every i, when the number X_i is positive, the player receives from the casino a sum of money equal to X_i. When X_i is negative, the player pays the casino a sum of money equal to $|X_i|$.

(d) What is the standard deviation of the net total gain of a player after n plays of the Gaussian slot machine?

(e) What is the probability that the absolute value of the net total gain after n plays is greater than $2\sqrt{n}\sigma$?

2. People who wish to use a particular mailbox arrive at the box in a Poisson manner with average arrival rate of λ customers per hour. Independently, each user of the mailbox wishes to mail either one letter (with probability $\frac{2}{3}$) or one parcel (with probability $\frac{1}{3}$). Although the users arrive one at a time in a Poisson manner, the ith user is accompanied by N_i non-user friends, where the N_i's are independent and identically-distributed random variables with an associated transform

$$M_N(s) = \frac{1}{2} + \frac{1}{3}e^s + \frac{1}{6}e^{2s} \qquad .$$

(a) Determine the expected value of Y, the total number of people arriving at the mailbox during a three hour interval.

(b) At 3 P.M. today, we shall begin counting letters and parcels as they are brought to the mailbox. What is the probability we shall see a total of exactly three parcels by the time the fifth letter arrives?

(c) Determine the probability that exactly three out of the next eight users will mail parcels.

(d) If we make an equally likely selection from all people (users and non-users) who arrive at the mailbox in the last week, what is the probability we select a person who was accompanied when he or she arrived at the mailbox

(e) Determine either the PMF or the transform for K, the total number of people arriving at the mailbox during a particular hour.

(f) For a time selected by random incidence, determine the transform for T, the total time interval from the arrival of the third previous user until the arrival of the fifth future user of the mailbox.

3. Mary loves gambling. She starts out with \$200. She can bet either \$100 or \$200 (assuming she has sufficient funds), and wins with probability p.

(a) Assuming that she stops when she runs out of money or when she has reached \$400, what is the optimal betting strategy? (i.e., how much should she bet when she has \$100, \$200, \$300? The amount she bets does not have to be the same amount at each time.)

(b) What is the expected number of transitions until she either runs out of money or reaches \$400 for $p = 0.75$ under the optimal strategy?