# LECTURE 15

## The Bernoulli process

- **Readings:** Section 5.1

## Lecture outline

- Definition of Bernoulli process
- Random processes
- Basic properties of Bernoulli process
- Distribution of interarrival times
- The time of the $k$th success

## The Bernoulli process

- A sequence of independent Bernoulli trials
- At each trial:
  - $\mathbf{P}(\text{success}) = \mathbf{P}(X = 1) = p$
  - $\mathbf{P}(\text{failure}) = P(X = 0) = 1 - p$
- Examples:
  - Sequence of ups and downs of the Dow Jones
  - Sequence of lottery wins/losses
  - Arrivals (each second) to a bank

## Random processes

- First view: sequence of random variables $X_1, X_2, \ldots$
- $\mathbf{E}[X_t] =$
- $\text{Var}(X_t) =$
- Second view: what is the right sample space?
- $\mathbf{P}(X_t = 1 \text{ for all } t) =$

## Number of successes $S$ in $n$ time slots

- $\mathbf{P}(S = k) =$
- $\mathbf{E}[S] =$
- $\text{Var}(S) =$

## Interarrival times

- $T_1$: number of trials until first success
  - $\mathbf{P}(T_1 = t) =$
  - Memoryless property
  - $\mathbf{E}[T_1] =$
  - $\mathsf{Var}(T_1) =$
  - If you buy a lottery ticket every day, what is the distribution of the length of the first string of losing days?

## Time of the $k$th arrival

- $Y_k$: number of trials to $k$th success
  - $\mathbf{E}[Y_k] =$
  - $\mathsf{Var}(Y_k) =$
  - $\mathbf{P}(Y_k = t) =$