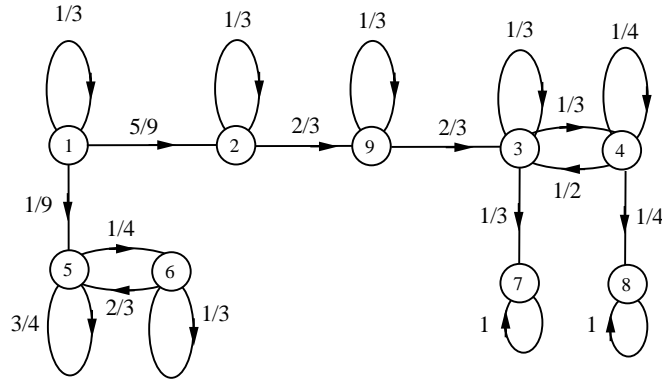


Tutorial 10
Week of April 18, 2005

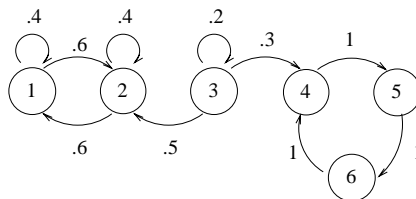
1. Consider a Markov chain X_n described by the diagram below.



- (a) What are the recurrent classes?
- (b) Find $\mathbf{P}(X_2 = 8 \mid X_0 = 3)$.
- (c) Assuming that $X_0 = 2$, what is the probability that the Markov chain eventually gets to state 7?
- (d) Find an approximation to $\mathbf{P}(X_{10000} = 5 \mid X_0 = 1)$.
- (e) Find an approximation to $\mathbf{P}(X_{10000} = 5 \mid X_0 = 5, X_{10001} = 6)$.
- (f) Suppose that $X_0 = 2$. Let T be the first time that state 3 is reached. What is the probability that $T = 10$?

For each of the following definitions of state X_k at time k ($k = 1, 2, \dots$), determine whether the Markov property is satisfied and, when it is, specify the transition probabilities p_{ij} :

2. (a) A six-sided die is rolled repeatedly.
 - i. Let X_k denote the largest number rolled in the first k rolls.
 - ii. At time k , let X_k be the number of rolls since the most recent six.
 - (b) Let Y_k be the state of some discrete-time Markov process at time k (i.e., it is known Y_k satisfies the Markov property) with known transition probabilities q_{ij} .
 - i. For a fixed integer $r > 0$, let $X_k = Y_{r+k}$.
3. The Markov chain shown below is in state 3 immediately before the first trial.



- (a) Indicate which states, if any, are recurrent, transient, and periodic.

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- (b) Find the probability that the process is in state 3 after n trials.
- (c) Find the expected number of trials up to and including the trial on which the process leaves state 3.
- (d) Find the probability that the process never enters state 1.
- (e) Find the probability that the process is in state 4 after 10 trials.
- (f) Given that the process is in state 4 after 10 trials, find the probability that the process was in state 4 after the first trial.