Recitation 23
May 10, 2005

1. Random variable X takes on experimental values of -8, 0, and 8 with probabilities of $\frac{1}{8}$, $\frac{6}{8}$, and $\frac{1}{8}$, respectively. T_n is the sum and A_n is the average of n independent experimental values of X (i.e., T_{100} is the sum of 100 independent experimental values of X.)

   (a) Evaluate the expectations and variances for T_n and A_n.

   (b) Is T_n convergent in probability? Is A_n convergent in probability? Give a full explanation in each case, and if the quantity is convergent in probability, also specify the limit to which it converges.

   (c) Provide an <u>excellent</u> numerical approximation for the quantity:

$$Q = \mathbf{P}\left(|T_{100} - E[T_{100}]| \geq 32\right)$$

2. Combine your understanding of Poisson processes and your understanding of approximations based on central limit theorems to determine the numerical values of a and b in the following approximation:

$$\left.\left(\frac{3^{64}y^{63}e^{-3y}}{63!}\right)\right|_{y=\frac{68}{3}} \simeq \frac{1}{\sqrt{2\pi}a}e^{-\frac{b^2}{2a^2}}$$

   Also, explain the justification for this approximation.

3. (Problem 7.7 from the textbook) During each day, the probability that your computer's operating system crashes at least once is 5%, independent of every other day. You are interested in the probability of at least 45 crash-free days out of the next 50 days.

   (a) Find the probability of interest by using the normal approximation to the binomial.

   (b) Repeat part (a), this time using the Poisson approximation to the binomial.