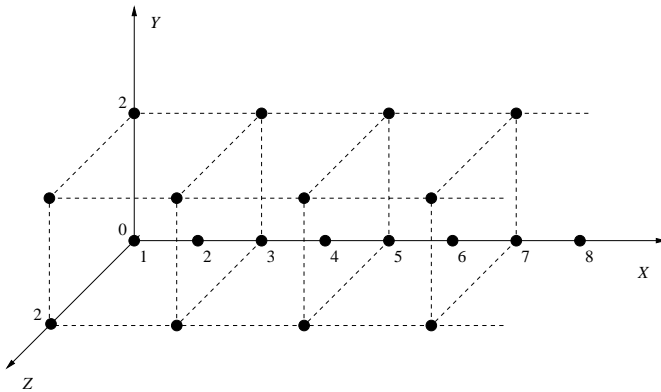


Tutorial 3
Week of February 21, 2005

1. Chuck will go shopping for probability books for K hours. Here, K is a random variable and is equally likely to be 1, 2, 3, or 4. The number of books N that he buys is random and depends on how long he shops. We are told that

$$p_{N|K}(n | k) = \frac{1}{k}, \quad \text{for } n = 1, \dots, k.$$

- (a) Find the joint PMF of K and N .
 - (b) Find the marginal PMF of N .
 - (c) Find the conditional PMF of K given that $N = 2$.
 - (d) We are now told that he bought at least 2 but no more than 3 books. Find the conditional mean and variance of K , given this piece of information.
2. Consider three random variables X , Y , and Z , associated with the same experiment. The random variable X is geometric with parameter p . If X is even, then Y and Z are equal to zero. If X is odd, (Y, Z) is uniformly distributed on the set $S = \{(0, 0), (0, 2), (2, 0), (2, 2)\}$. The figure below shows all the possible values for the triple (X, Y, Z) that have $X \leq 8$. (Note that the X axis starts at 1 and that a complete figure would extend indefinitely to the right.)



- (a) Find the joint PMF $p_{X,Y,Z}(x, y, z)$.
- (b) Answer with “yes” or “no” and one sentence of explanation:
 - (i) Are Y and Z independent?
 - (ii) Given that $Z = 2$, are X and Y independent?
 - (iii) Given that $Z = 0$, are X and Y independent?
 - (iv) Given that $Z = 2$, are X and Z independent?
- (c) Find $\text{var}(Y + Z | X = 5)$.