

6.041 Spring 2005 Quiz 1
Monday, March 7, 12:05-12:55 p.m.

DO NOT TURN THIS QUIZ OVER UNTIL
YOU ARE TOLD TO DO SO

- You have 50 minutes to complete the quiz.
- Write your solutions in the exam booklet. We will not consider any work not in the exam booklet.
- This quiz has two problems that are not necessarily in order of difficulty.
- You may give an answer in the form of an arithmetic expression (sums, products, ratios, factorials) of numbers that could be evaluated using a calculator. Expressions like $\binom{8}{3}$ or $\sum_{k=0}^5 (1/2)^k$ are also fine.
- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgment of your level of understanding as reflected by what you have written.
- This is a closed-book exam except for one double-sided, handwritten, 8.5 by 11 formula sheet plus a calculator.
- Be neat! If we can't read it, we can't grade it.
- At the end of the quiz, turn in your solutions along with this quiz (this piece of paper).

Write your name, your recitation instructor's name, and your TA's name on the front of the booklet. (3 points)

Problem 1: (27 points)

Consider the following game. You throw two fair coins independently. Let's refer to the coins as coin 1 and coin 2. If they have the same outcome, you win; otherwise, you lose.

- (a) (15 points) Is the outcome of coin 1 independent of whether you win or lose? Is the outcome of coin 2 independent of whether you win or lose? Explain your answer.
- (b) (12 points) Are the outcomes of the two coins independent of each other conditioned on your having won or lost? Explain your answer.

Problem 2: (70 points)

Your filthy roommate, Jerome, is (finally) doing his laundry. He has so much to do that he is doing it over four separate days—Monday, Tuesday, Wednesday, and Thursday—and, on each day, he has five quarters in each of his left and right pockets for use in the coin-operated machines. When drawing a quarter from his pockets, he draws from his left pocket with probability p and from his right pocket with probability $1 - p$, independently of previous draws.

On Monday, Jerome draws five quarters from his pockets to get the \$1.25 required for a washing machine. It turns out that one quarter is drawn from his left pocket and four quarters are drawn from his right pocket. To begin with, one of the quarters in his left pocket is Canadian (and all the other nine are American).

- (a) (10 points) What is the probability that his Canadian quarter remains in his left pocket?
- (b) (18 points) What is the expectation and variance of the number of American quarters remaining in his left pocket?

On Tuesday, Jerome again draws five quarters from his pockets.

- (c) (10 points) What is the probability that exactly three of the five quarters drawn are from his left pocket?

On Wednesday, Jerome decides to use the extra-large capacity washing machine, which costs \$2.00; so he draws eight quarters from his pockets.

- (d) (12 points) It turns out that the eighth quarter drawn is the fifth drawn from his left pocket, leaving it empty. What is probability of this event?

On Thursday, Jerome does not draw a fixed number of quarters from his pockets, but instead draws quarters until one of his pockets is empty. Let X be the number of quarters remaining in his other pocket.

- (e) (20 points) Let A be the event that Jerome's left pocket empties before his right pocket. Find $\mathbf{P}(X = x \cap A)$ for $x = 1, 2, \dots, 5$.