Recitation 2G Answers
February 10, 2005

3. That (a) implies (b) is trivial. Suppose then that (b) holds. Consider the outcomes numbered $i_1, i_2, \ldots, i_m$ and let $u_j \in \{H, T\}$ for $1 \le j \le m$. Let S_j be the set of all sequences of length $M = \max(i_j : 1 \le j \le m)$ showing u_j in the i_jth position. Clearly $|S_j| = 2^{M-1}$ and $|\cap_j S_j| = 2^{M-m}$. Therefore,

$$\mathbf{P}(S_j) = \frac{2^{M-1}}{2^M} = \frac{1}{2}, \quad \mathbf{P}\left(\cap_j S_j\right) = \frac{2^{M-m}}{2^M} = \frac{1}{2^m}, \tag{1}$$

so that $\mathbf{P}(\cap_j S_j) = \prod_j \mathbf{P}(S_j)$.

4. (a) Flip two coins; let A be the event that the first shows H, let B be the event that the second shows H, and let C be the event that they show the same. Then A and B are independent, but not conditionally independent given C.

   (b) Roll two dice; let A be the event that the smaller is 3, let B be the event that the larger is 6, and let C be the event that the smaller score is no more than 3, and the larger is 4 or more. Then A and B are conditionally independent given C, but not independent.

   (c) The definitions are equivalent if $\mathbf{P}(C) = 1$.