Recitation 12
March 29, 2005

1. (a) Imagine that you first roll a fair die and then you flip a fair coin the number of times shown by the die. Letting H denote the number of these flips that come up heads, find $\mathbf{E}[H]$ and $\text{var}(H)$.

   (b) Repeat part (1a) only now assume you first roll two dice.

2. A factory that produces widgets also packages them for shipment. Each particular shipment contains a number of crates, each particular crate contains a number of boxes, and each particular box contains a number of widgets. Let

   - X, the number of widgets in any particular box.
   - N, the number of boxes in any particular crate.
   - K, the number of crates in any particular shipment.

   X, N and K are independent indentically distributed non-negative random variables with expectation equal to 10 and variance equal to 16. Evaluate the expectation and variance for:

   (a) T, the number of widgets in any particular crate.

   (b) W, the total number of widgets in any particular shipment.

3. Let $X_1, X_2, \ldots$ be independent normal random variables with mean 2 and variance 4. Let N be a geometric random variable which is independent of the X_i, with parameter $p = 2/3$. (In particular, $\mathbf{E}[N] = 3/2$ and $\mathbf{E}[N^2] = 3$.)

   (a) If δ is a small positive number, we have $\mathbf{P}(|X_1| \leq \delta) \approx \alpha\delta$, for some constant α. Find the value of α. (Your answer may involve π, no need to evaluate numerically.)

   (b) Find $\mathbf{E}[X_1 N]$.

   (c) Find the variance of $X_1 N$.

   (d) Find $\mathbf{E}[X_1 + \cdots + X_N \mid N \geq 2]$.

   (e) Write down the transform associated with $N + X_1 + \cdots + X_N$.