Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2005)

Recitation 16
April 12, 2005
Review of Bernoulli and Poisson Processes

1. Answer *True* if the statement is *always* true; otherwise answer *False*.
   (a) Let $\{X_i\}$ be a Bernoulli process with parameter p and let $N_i = \sum_{j=1}^{i} X_j$ be its associated counting process.
      i. $\mathbf{P}(N_1 = N_2) = \mathbf{P}(N_3 = N_4)$.
      ii. Let $Y = \sum_{k=3}^{7} X_k$. Then Y is a Pascal random variable of order 5 with parameter p.
   (b) Let N_t be a Poisson process with rate $\lambda = 2$.
      i. Let $X = N_{3.2} - N_{1.7}$. Then $\text{var}(X) = \frac{1}{(3.2)^2}$.
      ii. Let $X = N_{3.2} - N_{1.7}$. Then X and N_4 are independent.
2. Problem 5 from Chapter 5 in Textbook - Random incidence in the Bernoulli process: You have a cousin who has been addicted to playing the same video game from time immemorial. Assume that he wins each game with probability p, independently of what happens in other games. You enter his room at some random time and find out that he's losing his current game.
   - What is the probability distribution of the number of lost games between two consecutive wins?
   - At the time you enter the room, what is the probability distribution of the number of lost games between his most recent win and his next future win?
   - How do your answers above compare? Explain.
3. The amount of time between arrivals in an arrival process is the interarrival time. If the interarrival times are independent and identically distributed positive random variables, we call the arrival process a renewal process. (Notice that a Poisson process of rate λ is a renewal process where the interarrival times are exponentially distributed with parameter λ.)

   Suppose bus arrivals at a bus stop form a renewal process where the interarrival times are uniformly distributed between 1 and 2 hours.

   (a) Find the PDF for the second-order interarrival time (i.e. the interarrival time between every other arrival).
   (b) Given that a bus arrived at 12pm, at what time do we expect the fourth bus which comes after 12pm to arrive?

   Suppose now that we have a renewal process where the interarrival times are uniformly distributed between 1 and 2 hours, and each arrival consists of either 1 or 2 buses. If the interarrival time immediately before the ith arrival is greater than 1.5 hours, then the probability that the ith arrival has 2 buses is $\frac{1}{2}$. If the interarrival time immediately before the ith arrival is less than 1.5 hours, then the ith arrival has 1 bus.

   (c) Given that a bus arrived at 12pm, at what time do we expect the fourth bus which comes after 12pm to arrive?

(d) Given that a bus arrived at 12pm, at what time do we expect the fourth 2-bus arrival which comes after 12pm to arrive?