# LECTURE 18

## Markov Processes – I

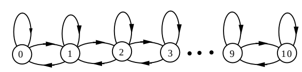
- **Readings:** Sections 6.1–6.2

## Lecture outline

- Checkout counter example
- Markov process definition
- $n$-step transition probabilities
- Classification of states

## Checkout counter model

- Discrete time $n = 0, 1, \ldots$
- Customer arrivals: Bernoulli($p$)
  - geometric interarrival times
- Customer service times: geometric($q$)
- "State" $X_n$: number of customers at time $n$




## Finite State Markov models

- $X_n$: state after $n$ transitions
  - belongs to a finite set, e.g., $\{1, \ldots, m\}$
  - $X_0$ is either given or random
- **Markov property/assumption:** (given current state, the past does not matter)

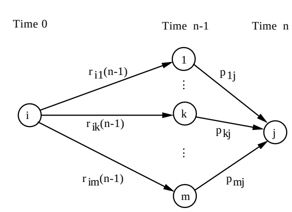
$$\begin{aligned} p_{ij} &= \mathbf{P}(X_{n+1} = j \mid X_n = i) \\ &= \mathbf{P}(X_{n+1} = j \mid X_n = i,\ X_{n-1}, \ldots, X_0) \end{aligned}$$

- Modeling steps:
  - identify the possible states
  - mark the possible transitions
  - record the transition probabilities

## $n$-step transition probabilities

- State occupancy probabilities, given initial state $i$:

$$r_{ij}(n) = \mathbf{P}(X_n = j \mid X_0 = i)$$



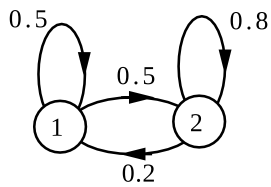

- Key recursion:

$$r_{ij}(n) = \sum_{k=1}^{m} r_{ik}(n-1) p_{kj}$$

- With random initial state:

$$\mathbf{P}(X_n = j) = \sum_{i=1}^{m} \mathbf{P}(X_0 = i) r_{ij}(n)$$

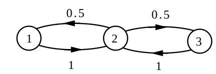
## Example




| | n=0 | n=1 | n=2 | n=2563 | n=2564 |
|---|---|---|---|---|---|
| $r_{11}(n)$ | | | | | |
| $r_{12}(n)$ | | | | | |

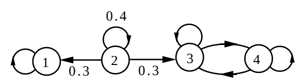
## Generic question:

- Does $r_{ij}$ converge to something?




n odd: $r_{22}(n)$= n even: $r_{22}(n)$=

- Does the limit depend on initial state?




$r_{11}(n)$=

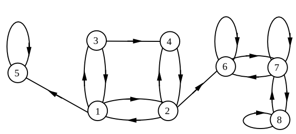
$r_{31}(n)$=

$r_{21}(n)$=

## Recurrent and transient states

- State $i$ is **recurrent** if:
  starting from $i$,
  and from wherever you can go,
  there is a way of returning to $i$

- If not recurrent, called **transient**




  - $i$ transient:
    $\mathbf{P}(X_n = i) \to 0$,
    $i$ visited finite number of times

- **Recurrent class**:
  collection of recurrent states that
  "communicate" to each other
  and to no other state