Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2005)

Tutorial 7
Week of March 28, 2005

1. This problem requires significant thought but essentially no calculations. Let X and Y be two random variables.

   Let

$$g(y) = \mathbf{E}[X|Y = y]$$
$$a = \mathbf{E}[(g(Y) - X)^2]$$

   Let

$$h(y) = \mathbf{E}[X] + \frac{\rho_{X,Y}\sigma_X}{\sigma_Y}(y - \mathbf{E}[Y])$$
$$= \mathbf{E}[X] + \frac{\text{cov}(X,Y)}{\sigma_Y^2}(y - \mathbf{E}[Y]).$$
$$b = \mathbf{E}[(h(Y) - X)^2].$$

   (a) Compare the magnitudes of a and b.

   (b) Suppose all we know about the random variables X and Y is their correlation coefficient $\rho_{X,Y}$. Specify all values of $\rho_{X,Y}$, if any, for which we can be sure that $b = 0$. Explain your reasoning.

2. Problem 4.13 from p.261 of the text.

   Pat and Nat are dating, and all of their dates are scheduled to start at 9 p.m. Nat always arrives promptly at 9 p.m. Pat is highly disorganized and arrives at a time that is uniformly distributed between 8 p.m. and 10 p.m. Let X be the time in hours between 8 p.m. and the time when Pat arrives. If Pat arrives before 9 p.m., their date will last exacty 3 hours. If Pat arrives after 9 p.m., their date will last for a time that is uniformly distributed between 0 and $3 - X$ hours. The date starts at the time they meet. Nat gets irritated when Pat is late and will end the relationship after the second date on which Pat is late by more than 45 minutes. All dates are independent of any other dates.

   (a) What is the expected number of hours Nat waits for Pat to arrive?

   (b) What is the expected duration of any particular date?

   (c) What is the expected number of dates they will have before breaking up?

3. Problem 4.18, page 262 in the text. At a certain time, the number of people that enter an elevator is a random variable with Poisson PMF, with parameter λ. The weight of each person to get on the elevator is independent of each other person's weight, and is uniformly distributed between 100 and 200 lbs. Let X_i be the fraction of 100 by which the i^{th} person exceeds 100 lbs, e.g. if the 7^{th} person weighs 175 lbs., then $X_7 = .75$. Let Y be a random variable given by the sum of the X_i.

   (a) What is the probability that at least one person to get on the elevator weighs more than 150 lbs.?

(b) Find the generating function for Y.

(c) Use (b) to find the expected value of Y.

(d) Use the conditional expectation to verify your answer above.