Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2005)

Tutorial 7 Solutions
Week of March 28, 2005

1. (a) $a \leq b$ for all possible values of X, Y. Since a is the mean squared estimation error of the least squares estimator of X based on Y, and b is the mean squared estimation error of the *linear* least squares estimator of X based on Y, we must have $a \leq b$, because removing the linearity constraint can only improve the optimization, not make it worse.

   (b) $\rho_{X,Y} = \pm 1$

   Explanation: Since $b = \sigma_X^2(1-\rho_{X,Y}^2)$, it is evident that the only way to pick $\rho_{X,Y}$ to get $b = 0$ is $\rho_{X,Y} = \pm 1$. For these two values of $\rho_{X,Y}$, we have $X = h(Y)$, so

$$X = \mathbf{E}[X] \pm \frac{\sigma_X}{\sigma_Y}(Y - \mathbf{E}[Y]) \ ,$$

   i.e., X and Y are linearly related.

2. See online solutions.

3. (a) The probability that there are k people in an elevator is given by:

$$P(k) = e^{-\lambda}\frac{\lambda^k}{k!}$$

   Let A denote the event that at least one person to get on the elevator weighs more than 150 pounds. The probability of A is 1 minus the probability that no one weighs more than 150 pounds.

$$\begin{aligned} P(A) &= 1 - \sum_{k=0}^{\infty} e^{-\lambda}\frac{\lambda^k}{k!}\left(\frac{1}{2}\right)^k \\ &= 1 - e^{-\lambda}\sum_{k=0}^{\infty}\frac{(\lambda/2)^k}{k!} \\ &= 1 - e^{-\frac{\lambda}{2}} \end{aligned}$$

   See textbook solutions online for the other parts.