

LECTURE 3

- **Readings:** Section 1.5
- Review
- Independence of two events
- Independence of a collection of events

Review

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad \text{assuming } P(B) > 0$$

- Multiplication rule:

$$P(A \cap B) = P(B) \cdot P(A | B) = P(A) \cdot P(B | A)$$

- Total probability theorem:

$$P(B) = P(A)P(B | A) + P(A^c)P(B | A^c)$$

- Bayes rule:

$$P(A_i | B) = \frac{P(A_i)P(B | A_i)}{P(B)}$$

Independence of two events

- **Defn:**

$$P(A \cap B) = P(A) \cdot P(B)$$

- Recall $P(A \cap B) = P(A) \cdot P(B | A)$

- Independence of B from A :

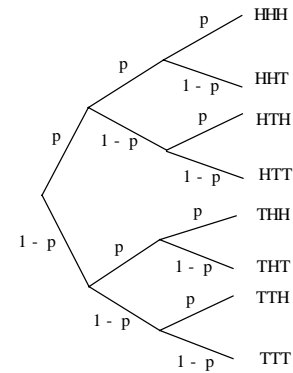
$$P(B | A) = P(B)$$

- By symmetry, $P(A | B) = P(A)$

Models based on conditional probabilities

- 3 tosses of a biased coin:

$$P(H) = p, P(T) = 1 - p$$



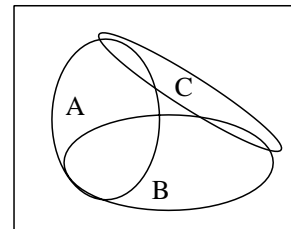
$$P(THT) =$$

$$P(1 \text{ head}) =$$

$$P(\text{first toss is H} | 1 \text{ head}) =$$

Conditioning may affect independence

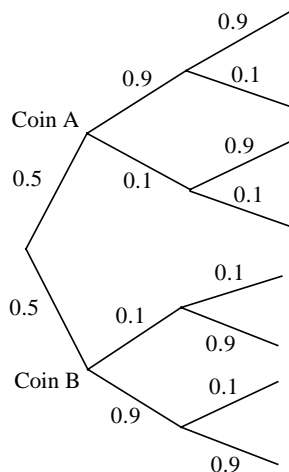
- Assume A and B are independent



- If we are told that C occurred, are A and B independent?

Conditioning may affect independence

- Two unfair coins, A and B :
 $P(H | \text{coin } A) = 0.9, P(H | \text{coin } B) = 0.1$
 choose either coin with equal probability



- Once we know it is coin A , are future tosses independent?

- If we do not know which coin it is, are future tosses independent?
 - Compare:
 - $P(\text{toss } 11 = H)$
 - $P(\text{toss } 11 = H | \text{first } 10 \text{ tosses are heads})$

Independence of a collection of events

- Intuitive definition:
 Information on some of the events tells us nothing about probabilities related to the remaining events
 - E.g.,

$$P(A_1 \cap (A_2^c \cup A_3) | A_5 \cap A_6^c) = P(A_1 \cap (A_2^c \cup A_3))$$
- Mathematical definition:
 For any distinct i, j, \dots, q ,

$$P(A_i \cap A_j \cap \dots \cap A_q) = P(A_i)P(A_j) \dots P(A_q)$$

Independence vs. pairwise independence

- Two independent fair coin tosses
 - A : First toss is H
 - B : Second toss is H
 - $P(A) = P(B) = 1/2$

HH	HT
TH	TT

- C : First and second toss have same outcome
 - $P(C) =$
 - $P(C \cap A) =$
 - $P(A \cap B \cap C) =$
 - $P(C | A \cap B) =$
- Pairwise independence **does not** imply independence

The king's sibling

- The king comes from a family of two children. What is the probability that his sibling is female?