Quiz 2 Announcements

Quiz 2: (closed-book; two *handwritten* double-sided 8.5x11 formula sheets and calculator permitted)

Date: Monday, April 11, 2005
Time: 12:05–12:55 p.m.

Content: All topics, with emphasis on those since Quiz 1, discussed in
Lectures 1 through 14
Textbook chapters 1 through 4
Recitations 1 through 13
Tutorials 1 through 7
Problem Sets 1 through 7

Optional Quiz Review Session: There will be two identical, two-hour quiz review sessions administered by the TAs. The sessions will consist of two parts. In the first hour, a concise overview of the theory will be presented. In the second hour, a set of practice problems will be solved. The quiz review is completely optional, but it is usually a good idea to attend and reinforce your understanding of the material, and perhaps gain some insight you did not have before. Details for the quiz review sessions are:

Date: Friday, April 8, 2005
Time: 4–6 p.m. and 6–8 p.m. (identical sessions)

The problems to be solved in the quiz review are attached. We strongly recommend working through the quiz review problems before coming to the quiz review.

Quiz 2 Review Problems

1. Random variables X and Y are independent and are described by the probability density functions $f_X(x)$ and $f_Y(y)$:

$$f_X(x) = \begin{cases} 1 & \text{if } 0 < x \leq 1, \\ 0 & \text{otherwise,} \end{cases} \qquad \text{and} \qquad f_Y(y) = \begin{cases} 1 & \text{if } 0 < y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

   Stations A and B are connected by two *parallel* message channels. One message from A to B is sent over each of the channels at the same time. Random variables X and Y represent the message delays in hours over parallel channels 1 and 2, respectively.

   A message is considered "received" as soon as it arrives on any one channel and it is considered "verified" as soon as it has arrived over both channels.

(a) Determine the probability that a message is received within 15 minutes after it is sent.

(b) Determine the probability that the message is received but not verified within 15 minutes after it is sent.

(c) Let T represent the time in hours between transmission at A and verification at B. Determine the CDF $F_T(t)$, and then differentiate it to obtain the PDF $f_T(t)$.

(d) If the attendant at B leaves for a 15-minute coffee break right after the message is received, what is the probability that he is present at the proper time for verification?

(e) The management wishes to have the maximum probability of having the attendant present for *both* reception and verification. Would they do better to let him take his coffee break as described above or simply allow him to go home 45 minutes after transmission?

2. The wombat club has N members, where N is a random variable with PMF

$$p_N(n) = p^{n-1}(1-p) \quad \text{for } n = 1, 2, 3, \ldots.$$

On the second Tuesday night of every month, the club holds a meeting. Each wombat member attends the meeting with probability q, independently of all the other members. If a wombat attends the meeting, then it brings an amount of money, M, which is a continuous random variable with PDF

$$f_M(m) = \lambda e^{-\lambda m} \quad \text{for } m \geq 0.$$

N, M, and whether each wombat member attends are all independent. Determine:

(a) The expectation and variance of the number of wombats showing up to the meeting.

(b) The probability that only one wombat shows up to the meeting.

(c) The transform of the PDF for the total amount of money brought to the meeting.

3. Oscar's dog has, yet again, run away from him. But, this time, Oscar will be using modern technology to aid him in his search. Oscar knows that X, the distance between him and his dog, is a random variable with a uniform distribution between five and ten miles. Oscar decides to use his pocket GPS device to help him pinpoint the dog's location.

Unfortunately, Oscar bought a cheap device and the measurement that Oscar receives is noisy, where the additive noise W, assumed to be independent of the actual location of the dog, is modeled as uniformly distributed between negative one and one. The measurement that Oscar reads on his display is random variable

$$Y = X + W \quad .$$

(a) Determine an estimator $g(Y)$ of X that minimizes $\mathbf{E}[(X - g(Y))^2]$ for all possible measurement values y. Provide a plot of this optimal estimator as a function of y.

(b) Determine the *linear* least squares estimator of X based on Y. Plot this estimator and compare it with the estimator from part (a). (For comparison, just plot the two estimators on the same graph and make some comments.)