

**Review problems for Chapter 7**

**Due: no due date**

1. Let  $X_1, X_2, \dots$  be independent, identically distributed, continuous random variables with  $E(X) = 2$  and  $\text{var}(X) = 9$ . Define  $Y_i = (0.5)^i X_i$ ,  $i = 1, 2, \dots$ . We also define  $T_n$  and  $A_n$  to be the sum and the average, respectively, of the terms  $Y_1, \dots, Y_n$ .
  - (a) Is  $Y_n$  convergent in probability? If so, to what value? Explain.
  - (b) Is  $T_n$  convergent in probability? If so, to what value? Explain.
  - (c) Is  $A_n$  convergent in probability? If so, to what value? Explain.
2. In your summer internship, you are working for the largest producer of lightbulbs. Your manager asks you to estimate the quality of the production; that is, to estimate  $p$ , the probability of a bulb produced by the factory to be defectless. You are told to assume that the quality of each bulb is independent, and identically distributed.
  - (a) Supposing you test  $n$  randomly picked bulbs, what is a good estimate for  $p$ ,  $Z_n$ , such that  $Z_n$  converges to  $p$  in probability?
  - (b) The management asks that the estimate is located in the range  $p \pm 0.1$  with probability 0.95. Are 27 randomly picked bulbs enough for this specification? Give the reason.
3. Imagine that on a particular roulette wheel,  $P(\text{WIN}) = \frac{18}{37}$ . If you play 100 games, find the probability that you win at least half of them. Hint: solve using CLT.
4. Problem 7.5 from the textbook (Page 403).