

Tutorial 5
Week of March 7, 2005

1. Dino, the cook, has good days and bad days with equal frequency. On a good day, the time (in hours) it takes Dino to cook a souffle is described by the PDF

$$f_G(g) = \begin{cases} 2, & \text{if } \frac{1}{2} < g \leq 1, \\ 0, & \text{otherwise,} \end{cases}$$

but on a bad day, the time it takes is described by the PDF

$$f_B(b) = \begin{cases} 1, & \text{if } \frac{1}{2} < b \leq \frac{3}{2}, \\ 0, & \text{otherwise.} \end{cases}$$

Find the conditional probability that today was a bad day, given that it took Dino less than three quarters of an hour to cook a souffle.

2. A point X is uniformly distributed on a line segment S iff it has a density function:

$$f_X(x) = \begin{cases} \gamma & x \in S \\ 0 & \text{otherwise} \end{cases}$$

Similarly, a vector is uniformly distributed on a region R iff it has a density function:

$$f_{XY}(x, y) = \begin{cases} \gamma & (x, y) \in R \\ 0 & \text{otherwise} \end{cases}$$

- a) Find γ .
- b) Show that if R is the square of side 1, centered at $(1/2, 1/2)$ then X, Y are independent, and are both uniformly distributed from $(0, 1)$.
- c) Show that for some R , X, Y need not be independent.
- d) For the R in (b) find the probability that the point (X, Y) lies in the circle inscribed in the square R .