

## LECTURE 2

- **Readings:** Sections 1.3-1.4

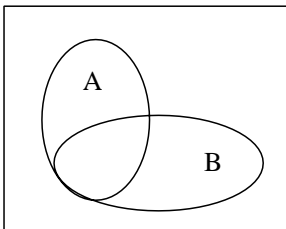
### Lecture outline

- Review
- Conditional probability
- Three **important** tools:
  - Multiplication rule
  - Total probability theorem
  - Bayes's rule

## Review of probability models

- Sample space
  - Mutually exclusive
  - Collectively exhaustive
  - Right granularity
- Allocation of probabilities to events
  1.  $P(A) \geq 0$
  2.  $P(\text{universe}) = 1$
  3. If  $A \cap B = \emptyset$ , then  $P(A \cup B) = P(A) + P(B)$
  - 3'. If  $A_1, A_2, \dots$  are disjoint events, then:  
 $P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \dots$
- Problem solving:
  - Setup sample space
  - Define probability law
  - Identify event of interest
  - Calculate...

### Conditional probability



- $P(A|B)$  = probability of  $A$ , given that  $B$  occurred
  - $B$  is our new universe

- **Definition:** Assuming  $P(B) \neq 0$ ,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

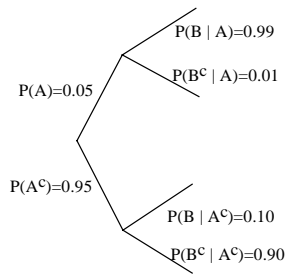
### Die roll example

	4				
Y = Second roll	3				
	2				
	1				
		1	2	3	4
		X = First roll			

- Let  $B$  be the event:  $\min(X, Y) = 2$
- Let  $M = \max(X, Y)$
- $P(M = 1 | B) =$
- $P(M = 2 | B) =$

## Models based on conditional probabilities

- Event  $A$ : Airplane is flying above
- Event  $B$ : Something registers on radar screen



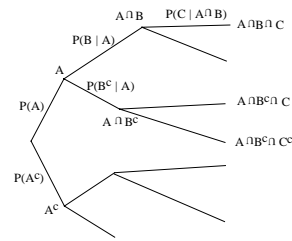
$$P(A \cap B) =$$

$$P(B) =$$

$$P(A | B) =$$

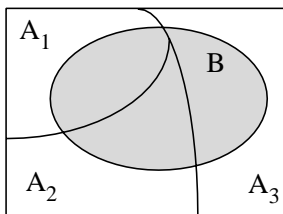
## Multiplication rule

$$P(A \cap B \cap C) = P(A)P(B | A)P(C | A \cap B)$$



## Total probability theorem

- Divide and conquer
- Partition of sample space into  $A_1, A_2, A_3$

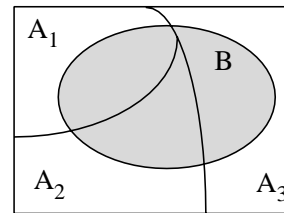


- One way of computing  $P(B)$ :

$$\begin{aligned} P(B) &= P(A_1)P(B | A_1) \\ &+ P(A_2)P(B | A_2) \\ &+ P(A_3)P(B | A_3) \end{aligned}$$

## Bayes's rule

- Rules for combining evidence
- "Prior" probabilities  $P(A_i)$
- We know  $P(B | A_i)$  for each  $i$
- Wish to compute  $P(A_i | B)$



$$\begin{aligned} P(A_i | B) &= \frac{P(A_i \cap B)}{P(B)} \\ &= \frac{P(A_i)P(B | A_i)}{P(B)} \\ &= \frac{P(A_i)P(B | A_i)}{\sum_j P(A_j)P(B | A_j)} \end{aligned}$$