

Problem Set 3

Due: February 23, 2005

1. A bank has a vault with a combination lock. The lock requires 8 different integers ranging from 1 and 90 to be entered in any order. However, the lock allows for 10 different integers ranging from 1 to 90 to be used as a combination; that is, the lock will open if any 8 of the 10 numbers in the combination are entered. Find the probability that a burglar will open the vault on the first try.
2. You are standing on Mass Ave, with a notepad, recording all the cars approaching MIT from Hahvahd bridge. The cars can either be a Compact or a non-Compact. By statistics, you know that the number of non-Compacts in Metro Boston is approximately 2 times that of Compacts.
 - (a) A non-Compact just passes by. What is the probability that the next car you see is also a non-Compact? What is the probability that you have to wait k more cars to see another non-Compact again? (i.e. You first see $k - 1$ Compacts, then followed by a non-Compact.)
 - (b) You have just observed a sequence of ten cars.
 - i. What is the probability that there were exactly one Compact and nine non-Compacts?
 - ii. How many distinct sequences of cars can be formed if the 10th car was also the 4th Compact you saw? What is the probability of each sequence?
 - iii. Using the result in part b ii), find the probability that the 10th car is the k th Compact you saw.
 - (c) You continue to record and gradually become bored. You decide that you will record 25 more cars. Find the number of all possible distinct car sequences under the following alternative assumptions.
 - i. All 25 cars are recorded.
 - ii. You stop when the number of either type of car reaches 13.
3. Each of n files is saved independently onto either hard drive A (with probability p) or hard drive B (with probability $1 - p$). Let A be the total number of files selected for drive A and let B be the total number of files selected for drive B.
 - (a) Determine the PMF, expected value, and variance for random variable A .
 - (b) Evaluate the probability that the first file to be saved ends up being the only one on its drive.
 - (c) Evaluate the probability that at least one drive ends up with a total of exactly one file.
 - (d) Evaluate the expectation and the variance for the difference, $D = A - B$.
 - (e) Assume $n \geq 2$. Given that both of the first two files to be saved go onto drive A, find the conditional expectation, variance and probability law for random variable A .
4. A particular class has had a history of low attendance. Fed up with the situation, the professor decides that she will not lecture unless at least k of the n students enrolled in the class are present. Each student will independently show up with probability p_g if the weather is good, and with probability p_b if the weather is bad. If the chance of bad weather tomorrow is $P(B)$ what is the probability that the professor teaches her class?

5. The president of a company discovers that one of her two vice presidents, A and B is embezzling money from the company. In order to determine who is guilty, she decides to hire a private detective to investigate. If she chooses to investigate VP A she will have to pay D_A to the detective, and if A turns out to be guilty, the president will have to pay R_A to replace A . Similarly, investigating B has costs D_B and R_B . Furthermore, if the detective decides that one of the VP's is innocent, the president will have to pay the detective to investigate the other VP. If the a priori probability that A is guilty is p , and that B is guilty is $1 - p$, find the conditions on p, D_A, D_B, R_A, R_B for which investigating A first would minimize the expected cost of the procedure.