

## LECTURE 4

- **Readings:** Section 1.6

### Lecture outline

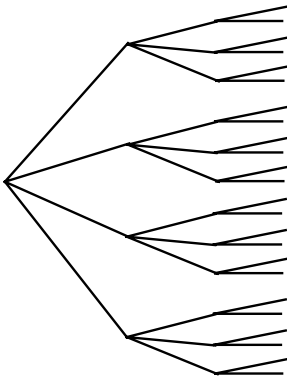
- Principles of counting
  - Many examples
- Binomial probabilities

## Discrete uniform law

- Let all sample points be equally likely
- Then,
$$P(A) = \frac{\text{number of elements of } A}{\text{total number of sample points}}$$
- Just count...

### Basic counting principle

- $r$  steps
- $n_i$  choices at step  $i$



- Number of choices is  $n_1 n_2 \cdots n_r$
- Number of license plates with 3 letters and 4 digits =
- ... if repetition is prohibited =

- **Permutations:** Number of ways of ordering  $n$  elements is:
- Number of subsets of  $\{1, \dots, n\} =$

## Example

- Probability that six rolls of a six-sided die all give different numbers?
  - Number of outcomes that make the event happen:
  - Number of elements in the sample space:
  - Answer:

## Combinations

- $\binom{n}{k}$ : number of  $k$ -element subsets of a given  $n$ -element set
- Two ways of constructing an ordered sequence of  $k$  **distinct** items:
  - Choose the  $k$  items one at a time:  

$$n(n-1)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$
 choices
  - Choose  $k$  items, then order them ( $k!$  possible orders)
- Hence:

$$\binom{n}{k} \cdot k! = \frac{n!}{(n-k)!}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

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$$\sum_{k=0}^n \binom{n}{k} =$$

## Binomial probabilities

- $n$  independent coin tosses
  - $P(H) = p$
- $P(HTTTHHH) =$
- $P(\text{sequence}) = p^{\# \text{ heads}}(1-p)^{\# \text{ tails}}$

$$\begin{aligned} P(k \text{ heads}) &= \sum_{k\text{-head seq.}} P(\text{seq.}) \\ &= (\# \text{ of } k\text{-head seqs.}) \cdot p^k(1-p)^{n-k} \\ &= \binom{n}{k} p^k(1-p)^{n-k} \end{aligned}$$

## Coin tossing problem

- event  $B$ : 3 out of 10 tosses were "heads".
  - What is the (conditional) probability that the first 2 tosses were heads, given that  $B$  occurred?
- All outcomes in conditioning set  $B$  are equally likely:  
 probability  $p^3(1-p)^7$ 
  - Conditional probability law is uniform
- Number of outcomes in  $B$ :
- Out of the outcomes in  $B$ , how many start with HH?

## Partitions

- 52-card deck, dealt to 4 players
- Find  $P$ (each gets an ace)
- Count size of the sample space  
(possible combination of "hands")

$$\frac{52!}{13! 13! 13! 13!}$$

- Count number of ways of distributing the four aces:  $4 \cdot 3 \cdot 2$
- Count number of ways of dealing the remaining 48 cards

$$\frac{48!}{12! 12! 12! 12!}$$

- Answer:

$$\frac{4 \cdot 3 \cdot 2 \frac{48!}{12! 12! 12! 12!}}{\frac{52!}{13! 13! 13! 13!}}$$